The Role of Financial Market Structure and the Trade Elasticity for Monetary Policy in Open Economies *

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(preliminary, comments welcome)

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Abstract

The degree of international risk sharing matters for how monetary policy should optimally be conducted in an open economy. This is because risk sharing affects the way in which monetary policy is affected by terms of trade considerations. In a standard twocountry model with monopolistic competition and nominal rigidities I consider different assumptions on international financial markets -complete markets, financial autarky and a bond economy- and a large region for the crucial parameter of the trade elasticity.

There are three main results: one, the prescription of (producer) price stability as the optimal policy is obtained only as a special case, while in general it is optimal to deviate from a strictly zero inflation rate. Two, while gains from international policy coordination are generally small, they do become substantial when international risk sharing is poor and wealth effects from shocks across countries are large. And, three, when international financial markets are incomplete, there are also (sometimes considerable) gains over the flexible price allocation achievable.

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1 Introduction

This paper presents a unifying framework in which to study the role of the degree of international risk sharing for optimal monetary policy in open economies. The main results are threefold: one, the prescription of (producer) price stability as the optimal policy is obtained only as a special case, while in general it is optimal to deviate from a strictly zero inflation rate. Two, while gains from international policy coordination are generally small, they do become substantial when international risk sharing is poor and wealth effects from shocks across countries are large. And, three, when international financial markets are incomplete, there are also (sometimes considerable) gains over the flexible price allocation achievable.

The framework used to address these issues is a relatively standard two-country imperfectlycompetitive sticky-price model of the open economy. In such a setup, a number of imperfections characterize the economy that typically exert influence on the way monetary policy should optimally be conducted. As in the closed economy both countries are characterized by two internal distortions: because of monopolistic competition output is inefficiently low, and the economy's adjustment is due to nominal rigidities as firms face quadratic price adjustment costs when they set their prices (assumed to be set in producer currency). In addition, there is an external distortion which stems from a country's monopoly power on the relative price of their exports to imports, that is, on its own terms of trade (TOT).¹ The present paper shows that this international dimension of monetary policy, that is, the size and direction in which movements in the terms of trade enter into the consideration of monetary policy, depends crucially on the degree of international risk sharing. The degree of risk sharing in turn, is determined by the structure of international financial markets assumed, as well as by the precise parameterization of the trade elasticity. For this purpose, I contrast three stylized assumptions on the international financial market structure - namely, complete markets, financial autarky and an incomplete markets-bond economy- and consider a wide range of the trade elasticity where domestic and foreign consumption goods are allowed to be substitutes or complements. The latter is of importance as it governs the strength in which relative wealth is affected in response to country specific shocks. Generally speaking, the stronger the wealth effects, the stronger the incentive of the policymaker to consider terms of trade movements in its optimal policy. In the special case of a unit trade elasticity (combined with log-utility) the terms of trade provide full 'automatic' risk sharing as any income effects from shocks are offset by proportional movements in the international relative price (see, Cole and Obstfeld (1991), Corsetti and Pesenti (2001)).

I study optimal monetary policy using a Ramsey approach and assuming that policymakers can commit. Gains from international policy coordination are computed by contrasting welfare achieved by a world social planner from the welfare achieved under uncoordinated (Nash) policy competition. On the methodological side, the present paper contributes to the literature by laying out a well defined Nash policy game in the framework of the Ramsey approach. In the following, I briefly discuss each of the three results found and try to position and contrast it with respect to the literature.

Turning to the first result in more detail, I find that for almost all cases of financial market assumptions and policy regimes considered the implications for monetary policy are

¹Throughout the paper, I refer the terms of trade when talking about the influence of international prices on policy decisions. However, it should be noted that, equivalently, I could have referred to the real exchange rate as the relevant international price to consider (which in this model moves always proportionally to the terms of trade).

that deviations from full producer price stability are optimal. Only in the case of complete markets and coordination and in the case of the unit-elasticity-'automatic perfect risk sharing' producer prices should be kept fully stabilized.² The deviations from price stability are brought about by considerations about the optimal variability of international prices, such as the terms of trade or the real exchange rate. In a related recent contribution De Paoli (2009) studies the role of financial market structure on optimal monetary policy in a small open economy in the form of comparing different targeting rules. In particular, she computes a welfare based ranking of producer price inflation targeting, consumer price inflation targeting, and a fixed exchange rate regime. The general setup of the present paper allows to uncover the precise patterns when it is optimal to fully stabilize, understabilize or overstabilize producer prices, together with the patterns of the corresponding optimal interest rate and terms of trade behavior.

In response to a productivity increase in, say, the domestic economy, I find that under *complete markets* (producer) price stability is always optimal when policymakers coordinate. When policymakers act uncoordinately, they follow a policy that results in a terms of trade responds by less than under flexible prices when goods are substitutes, as this would seem to allow them to achieve the same utility from consumption but at a lower disutility from labor effort. Under a Nash, however, both countries behave in this fashion and the terms of trade does no longer provide full risk sharing, leading to welfare losses. Conversely, when goods are complementary in consumption, the uncoordinated policy produces terms of trade flucuations which are too high relative to the efficient flexible price allocation.

When financial markets are incomplete (financial autarky or incomplete markets-bond case) policymakers, coordinated or uncoordinated, have to consider an additional distortion in their design of optimal policy. The TOT is found to be more even more depreciated (compared to a flexible price scenario) when goods are substitutes, which is exactly the opposite behavior as the optimal policy under complete markets. If policymakers now were to reduce employment, this would still benefit agents by increasing the utility of leisure; however, unlike under complete markets, consumption risk is not shared and consumption is much more closely linked to current output. As such, policymakers find it optimal to expand output so much when productivity is temporarily higher that the terms of trade depreciate even more that when compared to a flexible price world. The prescription of optimal policy flips again when moving to the region of complementary goods: in that case the TOT is found to be more appreciated relative to the flexible price response and producer price inflation decreases in response to a productivity shock.

In addition, the two-country setup allows for an explicit consideration of Nash versus coordinated optimal policies, which allows to also draw conclusions on the gains from policy cooperation. These are found increasing for elasticities of substitution away from unity and are typically an order of magnitude larger in the case of complementarity between domestic and foreign goods, particularly when risk sharing is low. In addition, I find that welfare gains from coordination are bigger under complete markets when goods are substitutes, but turn out to be bigger under financial autarky/ incomplete markets when goods are complements. This is due to the fact that the lack of risk sharing becomes even more important when, because of a low elasticity, international prices move strongly and wealth effects are large. Sutherland (2004) has previously studied the role of market structure for the gains of policy

²This is the 'isomorphism' and 'inward-looking' result of early contributions (see, e.g., Clarida, Gali and Gertler (2001), Gali and Monacelli (2005), or Corsetti and Pesenti (2001), Benigno and Benigno (2002)).

coordination in a simple static model, finding that welfare gains are rather small. In addition he only considers the case when goods are substitutes. On the one hand welfare gains are likely to be larger under more realistic, dynamic price rigidities³, on the other hand the case of low substitution elasticities is of particular interest as it precisely the specification needed to reconcile the predictions of the theoretical model with the low degree of risk sharing observed in the data.

It is interesting to note, that even if policymakers act coordinately price stability is, in general, not found to be the optimal outcome. The reason for this finding is that the flexible price financial autarky economy is not efficient as countries do not involve in any risk sharing. A policymaker that can, because of sticky prices, influence the terms of trade/ the real exchange rate finds it optimal to let it respond more closely to the way it would under complete financial markets, such that the equilibrium responses of the real exchange rate under the optimal policy is also doing some risk sharing. A related point is made recently by Corsetti, Dedola and Leduc (2009).

The rest of the paper is organized as follows. Section 2 describes the model and sets up the Ramsey problems for policymakers that act either under coordination or independently. Section 3 discusses the results for optimal monetary policy depending on the degree of intratemporal elasticity and depending on the financial market structure, looking at the implications for the optimality of price stability and the gains from policy coordination. Section 4 concludes.

2 The Model

The world economy consists of a Home country (H) and a Foreign country (F), each of which is specialized in one type of tradable good. Households and firms are defined over a continuum of unit mass. Home and Foreign households are indexed by $j \in [0, 1]$ and $j^* \in [0, 1]$ respectively. Each good is produced by firms in a number of varieties, indexed by h in the Home country and by f in the Foreign country. Each variety is an imperfect substitute to all other varieties and is produced under conditions of monopolistic competition. Firms face quadratic adjustment costs in their price setting decision and are assumed to set the price in the foreign market in their own currency (producer currency pricing). I abstract from modeling monetary frictions by considering a cashless economy. Unless necessary otherwise, in the following I only discuss the problem of Home agents, with an understanding that the problem for Foreign agents is symmetric - variables of Foreign agents are marked with an asterisk.

2.1 Model Setup

2.1.1 Preferences and Budget Constraint

Household j maximizes her lifetime expected utility:

 $^{^{3}}$ Faia and Monacelli (2004) also study the role of the terms of trade on optimal policy in a model with Rotemberg adjustment costs, but only consider the case of complete markets.

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}(j)}{1-\sigma} - \frac{L_t^{1+\kappa}(j)}{1+\kappa} \right\}$$
(1)

where β is the discount factor, C(j) is consumption and L(j) is labor effort. Consumption C(j) is a constant-elasticity-of-substitution (CES) basket over domestic and foreign goods:

$$C_t(j) = \left[\gamma^{\frac{1}{\omega}} C_{H,t}^{\frac{\omega-1}{\omega}}(j) + (1-\gamma)^{\frac{1}{\omega}} C_{F,t}^{\frac{\omega-1}{\omega}}(j)\right]^{\frac{\omega}{\omega-1}},$$
(2)

where ω denotes the trade elasticity, that is, the intratemporal elasticity of substitution between domestic and foreign goods, and where parameter $\gamma \geq \frac{1}{2}$ is the degree of home bias in consumption. For each household j the consumption indices of Home varieties and Foreign varieties are defined as:

$$C_{H,t} = \left[\int_{0}^{1} C_t \left(h,j\right)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t} = \left[\int_{0}^{1} C_t \left(f,j\right)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}$$
(3)

where $C_t(h, j)$ and $C_t(f, j)$ are respectively consumption of Home variety h and Foreign variety f by agent j at time t.

Household j maximizes equation (1) subject to the budget constraint. Each period household j receives wage income, $W_t L_t(j)$, and dividends from the monopolistic firms they own, $\Pi_t(j)$, and has consumption expenditure $P_t C_t(j)$. The availability of any assets of domestic household j depends on the assumptions of the structure of international financial markets. Throughout the paper, I consider three possible scenarios: complete markets, financial autarky and an incomplete markets bond economy.

Under complete markets the household has access to a full set of state-contingent (Arrow-Debreu) securities. Let $Q(s_{t+1}|s_t)$ denote the price of one unit of Home currency delivered in period t + 1 contingent on the state of nature at t + 1 being s_{t+1} . With complete markets, $Q(s_{t+1}|s_t)$ is the same for all individuals. Let $B_{H,t}(j, s_{t+1})$ denote the claim to $B_{H,t}(j, s_{t+1})$ units of Home currency at time t + 1 in the state of nature s_{t+1} , that household j buys at time t and brings into time t + 1. $Q^*(s_{t+1}|s_t)$ and $B_{F,t}(j, s_{t+1})$ are defined similarly in terms of units of Foreign currency. ε_t denotes the nominal exchange rate (units of Home currency per unit of Foreign currency). The budget constraint under complete markets is then given by:

$$\sum_{s_{t+1}} Q(s_{t+1}|s_t) B_{H,t}(j, s_{t+1}) + \sum_{s^{t+1}} Q^*(s_{t+1}|s_t) \varepsilon_t B_{F,t}(j, s_{t+1})$$

$$\leq B_{H,t-1}(j, s_t) + \varepsilon_t B_{F,t-1}(j, s_t) + W_t L_t(j) + \Pi_t(j) - P_t C_t(j)$$
(4)

If the two economies are in *financial autarky* no assets can be traded internationally. Let $B_{H,t}(j)$ and $B_{F,t}(j)$ denote bonds denominated in either domestic and foreign currency. Under international financial autarky, the domestic currency bond, $B_{H,t}$, that can be traded only domestically. Equivalently, foreign agents can trade a foreign currency bond, $B_{F,t}^*$, but also only within their country.⁴ The budget constraint of domestic household j under financial autarky then becomes:

$$B_{H,t}(j) = B_{H,t-1}(j) R_{t-1} + W_t L_t(j) + \Pi_t(j) - P_t C_t(j)$$
(5)

Finally, I consider the case of the *incomplete markets-bond economy*. We now assume that both countries can now engage in financial trade through one of the one-period nominal bonds. In particular, I assume that the foreign currency denominated bond, $B_{F,t}$, can be traded internationally (and net foreign wealth is initially zero).⁵ Following Schmitt-Grohe and Uribe (2003) and Benigno (2001), to render the incomplete markets economy stationary, I assume that domestic agents face a quadratic adjustment cost when taking on an international asset position different from their long-run (zero) position.⁶ The budget constraint under the assumption of the incomplete markets-bond economy is:

$$B_{H,t}(j) + \varepsilon_t B_{F,t-1}(j) + \frac{\phi}{2} \left(\frac{\varepsilon_t B_{F,t-1}(j)}{P_t} \right)^2 P_t$$

$$\leq B_{H,t-1}(j) R_{t-1} + \varepsilon_t B_{F,t-1}(j) R_{t-1}^* + W_t L_t(j) + \Pi_t(j) - P_t C_t(j)$$
(6)

2.1.2 Households' Intratemporal Consumption Allocation

Household j minimizes, each period, its consumption expenditure subject to obtaining a unit of the final consumption good. Denoting with P_t the Lagrange multiplier to that problem⁷ gives the following optimal demand functions:

$$c_t(h,j) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}(j) = \gamma \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} C_t(j),$$
(7)

$$c_t(f,j) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}(j) = (1-\gamma) \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\theta} \left(\frac{P_{F,t}}{P_t}\right)^{-\omega} C_t(j),$$
(8)

For given Home-currency prices of varieties, $p_t(h)$ and $p_t(f)$ the utility-based CPI, P_t , is given by:

⁴That is, $\int_{0}^{1} B_{H,t}(j) dj = 0$ and $\int_{0}^{1} B_{F,t}^{*}(j^{*}) dj^{*} = 0$.

⁵The nominal bonds are in zero net-supply worldwide, so that:

$$\int_{0}^{1} B_{H,t}(j) \, dj = 0, \int_{0}^{1} B_{F,t}(j) \, dj + \int_{0}^{1} B_{F,t}^{*}(j^{*}) \, dj^{*} = 0$$

 6 It is important to note that the internationally traded asset is exogeneously restricted to be the foreign currency bond only, for which a long-run zero position is simply assumed. In particular, this setup does not enter the recent literature on issues of portfolio choice and endogenous non-zero positions (see, e.g. Devereux and Sutherland (2008) and Tille and van Wincoop (2007)).

⁷Formally,

$$\min \int_{0}^{1} p_{t}(h) C_{t}(h, j) dh + \int_{0}^{1} p_{t}(f) C_{t}(f, j) df - P_{t}C_{t}(j).$$

$$P_{t} = \left[\gamma P_{H,t}^{1-\omega} + (1-\gamma) P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}},$$
(9)

where

$$P_{H,t} = \left[\int_{0}^{1} p_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, P_{F,t} = \left[\int_{0}^{1} p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}.$$
 (10)

2.1.3 Households' Labor Supply and Intertemporal Allocation

Denote with $\lambda_t(j)$ the Lagrange multiplier of the household's budget constraint. Household j's first order conditions with respect to $C_t(j)$ and $L_t(j)$ are identical for all possible financial market assumptions and are given by:

$$P_t C_t^{\sigma}(j) L_t^{\kappa}(j) = W_t \tag{11}$$

Under *complete financial market*, the first order condition w.r.t. home and foreign Arrow-Debreu securities are given by:

$$Q(s_{t+1}|s_t) = \beta E_t \left\{ \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}, Q^*(s_{t+1}|s_t) = \beta E_t \left\{ \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{\varepsilon_{t+1}}{\varepsilon_t} \right\}$$
(12)

which can be combined to obtain the risk sharing equation:

$$\frac{\varepsilon_t P_t^*}{P_t} = \left(\frac{C_t^*(j)}{C_t(j)}\right)^{-\sigma} \tag{13}$$

under $financial\ autarky$ the domestic currency bond can only be held domestically such that

$$1 = \beta E_t \left\{ R_t \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$
(14)

under the *incomplete markets bond economy*, the first order condition w.r.t. home and foreign bond are similarly given by:

$$1 = \beta E_t \left\{ R_t \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}, 1 = \beta E_t \left\{ R_t^* \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{\varepsilon_{t+1}}{\varepsilon_t} \right\}$$
(15)

The nominal interest rate R_t and R_t^* can be thought of as the underlying instruments of monetary policy in the two economies.

2.1.4 Production and Price Setting

The production function is assumed to be linear in labor:

$$Y_t(h) = Z_t L_t(h) \tag{16}$$

where Z_t is the level of productivity, which is given by a country-specific AR(1) process with persistence parameter ρ_Z and standard deviation σ_Z . Firms operate under conditions of monopolistic competition taking into account the downward-sloping demand for their product and set prices to maximize their profit. They are assumed to set the prices in the foreign market in their own currency, that is, I consider the scenario of producer currency pricing (PCP). Firms are small, in the sense that they ignore the impact of their pricing and production decisions on aggregate variables and price indices. When firms set their prices they have to consider a quadratic adjustment cost, with parameter α measuring the degree of price stickiness:

$$\phi_t(h) = \frac{\alpha}{2} \left(\frac{p_t(h)}{p_{t-1}(h)} - 1 \right)^2$$
(17)

The presence of Rotemberg adjustment costs makes the firms' price setting dynamic, which introduces richer and arguably more realistic equilibrium dynamic effects of monetary policy than in a setup where prices are set one period in advance. The richer description of price stickiness is also likely to be more appropriate for quantitive welfare analysis. I assume throughout that the law of one price holds, such that for each variety h we have $\varepsilon_t p_t^*(h) = p_t(h)$. Each producer chooses its price $p_t(h)$ such as to maximize its total market value:

$$E_{t}\left\{\sum_{t=0}^{\infty}\Omega_{0,t}\left[p_{t}\left(h\right)\left(1+\tau\right)-MC_{t}\left(h\right)\right]\left[\left(\frac{p_{t}\left(h\right)}{P_{H,t}}\right)^{-\theta}\left(C_{H,t}+C_{H,t}^{*}\right)\right]-\frac{\alpha}{2}\left(\frac{p_{t}\left(h\right)}{p_{t-1}\left(h\right)}-1\right)^{2}P_{H,t}\right\},$$
(18)

where MC_t is the marginal cost that minimizes labor input, which is equal to all firms, $MC_t(h) = MC_t = W_t/Z_t$, $\Omega_{t,s}$ is the household's stochastic discount factor between time t and s, and where τ stands for a production subsidy that offsets the distortion from monopolistic competition.

2.1.5 Firms' Optimality Conditions

The firm's optimal price setting condition is derived as:

$$0 = \left[\left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \left(C_{H,t} + C_{H,t}^* \right) \right] \left(\frac{p_t(h)}{P_{H,t}} \right)^{-1} \left[\theta \frac{MC_t(h)}{P_{H,t}} - (\theta - 1)(1 + \tau) \right] -$$
(19)
$$\alpha \left(\frac{p_t(h)}{p_{t-1}(h)} - 1 \right) \frac{P_{H,t}}{p_{t-1}(h)} + E_t \Omega_{t,t+1} \alpha \left(\frac{p_{t+1}(h)}{p_t(h)} - 1 \right) \frac{p_{t+1}(h)P_{H,t+1}}{p_t^2(h)}$$

Parameter $\alpha = 0$ corresponds to the case of flexible prices, in which case the price is set as the a simple markup over current marginal costs. We focus our attention on a symmetric equilibrium where all domestic producers charge the same price, adopt the same technology and therefore choose the same demand for labor. This implies $p_t(h) = P_{H,t}$, $p_t^*(h) = P_{H,t}^*$, $L_t(h) = L_t$, $\Pi_t(j) = \Pi_t$.

2.1.6 Resource Constraints and Aggregate Budget Constraints

The resource constraint for each variety h and each variety f are given by:

$$Y_t(h) = \int_0^1 c_t(h, j) dj + \int_0^1 c_t(h, j^*) dj^* + \int_0^1 \phi_t(j) dj = C_{H,t} + C_{H,t}^* + \phi_t$$
(20)

$$Y_t^*(f) = \int_0^1 c_t(f,j)dj + \int_0^1 c_t(f,j^*)dj^* + \int_0^1 \phi_t^*(j^*)dj^* = C_{F,t} + C_{F,t}^* + \phi_t^*$$
(21)

Symmetry across all households j gives $C_t(j) = C_t$, $L_t(j) = L_t$, $\lambda_t(j) = \lambda_t$, and implies that conditions (7)-(8), (11)-(13), (14) and (15) must also hold for aggregate variables and indices j can be dropped.

In addition, using equilibrium in the asset markets we can write the aggregate budget constraint under the case of financial autarky, having imposed clearing conditions⁸ as:

$$0 = W_t L_t + \Pi_t - P_t C_t \tag{22}$$

In the incomplete markets bond economy, after imposing asset market clearing conditions⁹ the budget constraint becomes:

$$\varepsilon_t B_{F,t} + \frac{\phi}{2} \left(\frac{\varepsilon_t B_{F,t-1}}{P_t} \right)^2 P_t = B_{F,t-1} R_{t-1}^* + W_t L_t + \Pi_t - P_t C_t$$
(23)

2.1.7 Relative Prices and The Terms of Trade

The terms of trade is defined as the price of imports to exports, $\frac{P_{F,t}}{\varepsilon_t P_{H,t}^*}$, which given the law of one price can be written as:

$$TOT_t = \frac{P_{F,t}}{P_{H,t}} \tag{24}$$

$$\int_{0}^{1} B_{H,t}(j) dj = 0 \text{ and } \int_{0}^{1} B_{F,t}^{*}(j^{*}) dj^{*} = 0.$$

⁹Under the szenario of the bond economy, these are given by:

$$\int_{0}^{1} B_{F,t}(j)dj = -\int_{0}^{1} B_{F,t}^{*}(j^{*})dj^{*}and \int_{0}^{1} B_{H,t}^{*}(j^{*})dj^{*} = 0.$$

⁸That is, under international financial autarky:

Using the optimal consumer price level resulting from the intratemporal allocation problem, it is possible to express all relative prices as a function of the terms of trade only. In particular, the real exchange rate, which is the price of a foreign consumption bundle relative to domestic consumption bundle, that is, $RER_t = (\varepsilon_t P_t^*)/P_t$, is related to the terms of trade by:

$$RER_{t} = f^{RER} \left(TOT_{t} \right) = \frac{\left[\gamma^{*} + (1 - \gamma^{*}) TOT_{t}^{1 - \omega} \right]^{\frac{1}{1 - \omega}}}{\left[\gamma + (1 - \gamma) TOT_{t}^{1 - \omega} \right]^{\frac{1}{1 - \omega}}}$$
(25)

The PPI-to-CPI ratios are defined as $p_{H,t} \equiv P_{H,t}/P_t$ and $p_{F,t}^* \equiv P_{F,t}^*/P_t^*$ and can also be written as functions of the terms of trade only:

$$p_{H,t} = f^{p_H} (TOT_t) = \left[\gamma + (1-\gamma) TOT_t^{1-\omega}\right]^{-\frac{1}{1-\omega}},$$

$$p_{F,t}^* = f^{p_F^*} (TOT_t) = \left[\gamma^* TOT_t^{\omega-1} + (1-\gamma^*)\right]^{-\frac{1}{1-\omega}}.$$
(26)

2.2 Optimal Monetary Policy and International Risk Sharing

Having completed the description of the model economy, we now turn to studying the optimal monetary policy in this two-country imperfectly competitive sticky price economy. For this reason, it is useful to first reflect on the distortions that characterize the economy. As in the closed economy both countries are characterized by two internal distortions: price stickiness and monopolistic competition. The latter produces an inefficient level of output. The other internal distortion is price stickingess which prevents efficient adjustment to the disturbances that affect the economy. A procyclical policy can remove the sticky-price distortion by making production supply-determined and can replicate the flex-price equilibrium if desirable. In addition, there is an external distortion which stems from a country's monopoly power on the international relative price, that is, on its own terms of trade. The strength and direction in which terms of trade considerations enter monetary policy crucially depends on the amount of international risk sharing, which in turn depends on a) the assumptions on asset markets and b) the degree of substitutability between domestic and foreign goods. This is because the effects of monetary policy in an open economy depend to a great extent on the influence it has on the nominal exchange rate, which in turn depends very much on the assumptions on the structure of international financial markets. On the other hand the degree of substitutability between goods produced in different countries determines the strength of the expenditure switching effect of exchange rate changes and therefore determines the impact of monetary policy on goods demand in different countries. Also, the elasticity influences the degree to which countries are subject to asymmetric income shocks. If the elasticity is close to unity then relative price changes are largely offset by changes in output volumes and the terms of trade provide strong automatic risk sharing (see Cole and Obstfeld (1991)).

To study the policy spillovers of international price movements and its influences on optimal monetary policy in isolation, I include a production subsidy in the model that offsets the distortion from monopolistic competition.

Further, I assume throughout that policymakers can credibly commit in the sense that thy can choose the entire future (state-contingent) evolution of the control variables, once and for all, at date zero. The assumption of commitment is important, as private sector expectations about the evolution of prices affect the forward looking terms in the dynamic pricing equations. I study these issues by employing a Ramsey type approach, following closely the steps outlined in Schmitt-Grohe and Uribe (2009) to obtain the steady state and dynamics implied by the Ramsey equilibrium. This builds on previous work on the study of optimal policy in dynamic economies, see e.g., (Ramsey (1927), Atkinson and Stiglitz (1976), Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991). In this setup the optimal monetary policy entails a Ramsey planner which maximizes a social objective function subject to the private sector's constraints.¹⁰ I compare optimal commitment policy under Nash competition and under cooperation, and compute welfare gains from coordinations from a second order approximation to the Ramsey equilibrium equations.

2.3 Definition of Equilibrium and Description of Constraints for Ramsey Problem

An equilibrium requires that households and firms behave optimally, as described by the above optimality conditions. Specifically, given exogenous processes for Z_t and Z_t^* , a policy for R_t and R_t^* and given initial conditions, a symmetric world competitive equilibrium is a set of prices and quantities that

- satisfy the Home and Foreign consumers' optimality conditions, equations (7)-(10), (11)-(??), (14) and their foreign counterparts, together with:
 - the risk sharing equation (13) under *complete financial markets*
 - equation (14) and the budget constraint, equation (23), under the *incomplete* markets-bond economy
 - the budget constraint, equation (22), under the *financial autarky*
- maximize firms profits, meaning that prices are set according to (19) and similarly in the foreign economy,
- satisfy the market clearing conditions for each asset and each good, in all the markets where it is traded, and
- satisfy the resource constraints.

It is possible to reduce the system of equilibrium conditions to a system of equations in $C_t, C_t^*, L_t, L_t^*, \pi_{H,t}, \pi_{F,t}^*$, and TOT_t only (given exogenous processes for Z_t and Z_t^* , and for a

¹⁰While most studies of optimal monetary policy in the recent literature build on a linear-quadratic approximation approach in the spirit of Rotemberg and Woodford (1997), Woodford (2003), and Benigno and Woodford (2004), recently, the Ramsey type approach has been employed in an increasing number of dynamic equilibium models with monopolistic competition and nominal rigidities. Examples include, in the context of closed economy models, Adao et. al (2003), Khan, King and Wolman (2003), Schmitt-Grohe and Uribe (2003, 2004), and Siu (2004). In the open economy a Ramsey-type approach has been employed by Faia and Monacelli (2003) and Arsenau (2004), open economy applications employing the linear-quadratic approach are, among others, Benigno and Benigno (2004) and De Paoli (2004).

policy for R_t and R_t^*). In particular, plugging in for the demand functions (7) and (8) together with their foreign counterparts, making use of the fact that $\pi_t = \frac{p_{H,t-1}}{p_{H,t}} \pi_{H,t}$, $\pi_t^* = \frac{p_{F,t-1}^*}{p_{F,t}^*} \pi_{F,t}^*$, and by using the functional relationships between the real exchange rate and the terms of trade (equation (25)) and the PPI-to-CPI ratio and the terms of trade (equation (26)), we can write the equilibrium as being described by equations (27)-(33a) below. Equations (27)-(28) are the two Euler equations, equations (29)-(30) the two price setting equations, equations (31)-(32) the two resource constraints, and optimality conditions (33a), (33b) or (33c) that hold under complete markets, financial autarky or the bond economy respectively.

$$1 = \beta E_t \left\{ R_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \right\}$$
(27)

$$1 = \beta E_t \left\{ R_t^* \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1}{\pi_{t+1}^*} \right\}$$
(28)

$$\alpha (\pi_{H,t} - 1) \pi_{H,t} = (p_{H,t})^{-\omega} [C_t + RER_t^{\omega} C_t^*] \left[\theta \left(\frac{L_t^{\kappa} C_t^{\sigma}}{Z_t p_{H,t}} \right) - (\theta - 1) (1 + \tau) \right]$$
(29)
+ $E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{p_{H,t+1}}{p_{H,t}} \alpha (\pi_{H,t+1} - 1) \pi_{H,t+1}$

$$\alpha \left(\pi_{F,t}^{*}-1\right) \pi_{F,t}^{*} = \left(p_{F,t}^{*}\right)^{-\omega} \left[RER_{t}^{-\omega}C_{t}+C_{t}^{*}\right] \left[\theta \left(\frac{L_{t}^{*\kappa}C_{t}^{*\sigma}}{Z_{t}^{*}p_{F,t}^{*}}\right) - (\theta-1)\left(1+\tau\right)\right]$$
(30)
+ $E_{t} \left(\frac{C_{t+1}}{C_{t}}\right)^{-\sigma} \frac{p_{F,t+1}^{*}}{p_{F,t}^{*}} \alpha \left(\pi_{F,t+1}^{*}-1\right) \pi_{F,t+1}^{*}$
 $Z_{t}L_{t} = (p_{H,t})^{-\omega} \left[C_{t}+RER_{t}^{\omega}C_{t}^{*}\right]$ (31)

$$Z_t^* L_t^* = \left(p_{F,t}^*\right)^{-\omega} \left[RER_t^{-\omega}C_t + C_t^*\right]$$
(32)

under complete markets:

$$RER_t = \left(\frac{C_t^*}{C_t}\right)^{-\sigma} \tag{33a}$$

under financial autarky:

$$p_{H,t}\left(Z_t L_t\right) - \phi_t = C_t \tag{33b}$$

under incomplete markets, bond economy¹¹:

$$(1 + \psi RER_t b_{F,t}) = \beta E_t \left\{ R_t^* \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}^*} \frac{RER_{t+1}}{RER_t} \right\}$$
(33c)
$$RER_t b_{F,t} + \frac{\phi}{2} \left(RER_t b_{F,t} \right)^2 = RER_t b_{Ft-1} \frac{R_{t-1}^*}{\pi_t^*} + p_{H,t} \left(Z_t L_t \right) - C_t - \phi_t$$

¹¹The budget constraint is also expressed in real terms, where $b_{F,t} = B_{F,t}/P_t^*$.

2.3.1 Definition of Ramsey problem under cooperation

To derive the Ramsey optimal monetary policy under cooperation, I set up the problem of a world social planner that aims at maximizing the country-sized weighted average measure of welfare, which are given by the lifetime expected utilities:

$$W_t^{average} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U\left(C_t, L_t\right) + U\left(C_t^*, L_t^*\right) \right\}$$
(34)

Under complete markets, define the vector of Ramsey constraints as RC_t^{CM} , by vertically stacking equations (27)-(33a). Superscript CM refers to the case of complete markets. Also, let superscript CMC refer to the case of complete markets and policymakers acting under coordination. I define the vector of Lagrange multipliers at time t attached to constraints (27)-(33a) by Λ_t^{CMC} , where $\Lambda_t^{CMC} = [\lambda_{H1,t}^{CMC}, \lambda_{F1,t}^{CMC}, \lambda_{F2,t}^{CMC}, \lambda_{H3,t}^{CMC}, \lambda_{F3,t}^{CMC}, \lambda_{4,t}^{CMC}]$. The portion of the Ramsey problem that is relevant for the purpose of computing the optimal policy from the timeless perspective is then given by:

$$L^{CMC} = \dots + \frac{1}{2} \left[U(C_t, L_t) + U(C_t^*, L_t^*) \right] + \frac{1}{2} \beta \left[U(C_{t+1}, L_{t+1}) + U(C_{t+1}^*, L_{t+1}^*) \right]$$
(35)
+
$$\frac{1}{\beta} \Lambda_{t-1}^{CMC} RC_{t-1}^{CM} + \Lambda_t^{CMC} RC_t^{CM} + \beta \Lambda_{t+1}^{CMC} RC_{t+1}^{CM}.$$

The optimal policy can then be described by the first order conditions of the Lagrangian with respect to Λ_t^{CMC} , and $C_t, C_t^*, L_t, L_t^*, \pi_{H,t}, \pi_{F,t}^*, TOT_t, R_t, R_t^*$.

Under financial autarky (FA) the Ramsey problem under cooperation can similarly be defined. Letting superscript FAC refer to the financial autarky scenario and coordinated policymakers, the vector of constraints RC_t^{FA} is given by vertically stacking equations (27)-(32) and (33b), and Λ_t^{FAC} is the vector of Lagrange multipliers to the constraints under FA. The relevant Lagrangian, L^{FAC} , is then obtained simply by replacing the constraints and Lagrange multipliers in equation (35) by Λ_t^{FAC} and RC_t^{FA} .

Finally, under the *incomplete markets bond economy* (*IM*) the Ramsey problem of coordinated policymakers can be summarized in a similar fashion by defining Λ_t^{IMC} and RC_t^{IM} (given by quations (27)-(32) and (33c)) accordingly. The incomplete markets-bond economy financial market structure now implies that the vector of constraints includes two financial market specific equations and the size of Lagrange multipliers is accordingly enlarged.

2.3.2 Definition of Ramsey problem under independently acting monetary authorities

If monetary authorities act uncoordinately, the home and foreign policymaker each maximize their respective national welfare, taking as given the other country's choice of policy. In particular, the objective of the domestic and the foreign policymaker are given by, respectively:

$$W_t = E_0 \sum_{t=0}^{\infty} \beta^t \{ U(C_t, L_t) \}$$
(36)

$$W_t^* = E_0 \sum_{t=0}^{\infty} \beta^t \{ U(C_t^*, L_t^*) \}$$
(37)

A Nash equilibrium is defined as a situation in which both countries choose the level of consumption, labor and inflation, and the policy instrument to maximize their objective, but where the other country's consumption, labor and inflation depend themselves on the policy choice of the first (not simply taken as given). As such, the home Ramsey policymaker should take as constraints its own Euler equation, pricing equation and resource constraint and financial market equilibrium condition as the relevant Ramsey constraints, *plus* the foreign consumption, labor and inflation choices as a function of its choice of the policy instrument. But these are nothing else than implicit functions defined by the foreign Euler equation, pricing equation and resource constraint. Therefore, each policymaker takes the entire competitive equilibrium conditions, given by equations (27)-(32) and the relevant financial market equilibrium equation, as constraints to the Ramsey policy problem.¹²

Under complete markets, the vector of constraints for the domestic and the foreign authority is given, as under coordination, by RC_t^{CM} . Also, define the sequence of the vector of Lagrange multipliers attached to constraints in RC_t^{CMN} and RC_t^{*CMN} by $\Lambda_t^{CMN} = [\lambda_{H1,t}^{CMN}, \lambda_{F1,t}^{CMN}, \lambda_{F2,t}^{CMN}, \lambda_{H3,t}^{CMN}, \lambda_{F3,t}^{CMN}, \lambda_{4,t}^{CMN}]$ and $\Lambda_t^{*CMN} = [\lambda_{H1,t}^{*CMN}, \lambda_{F1,t}^{*CMN}, \lambda_{H2,t}^{*CMN}, \lambda_{F3,t}^{*CMN}]$ and $\Lambda_t^{*CMN} = [\lambda_{H1,t}^{*CMN}, \lambda_{F1,t}^{*CMN}, \lambda_{H2,t}^{*CMN}, \lambda_{F2,t}^{*CMN}]$ and where superscript CMN refers to the scenario of complete markets and Nash policymakers. The relevant portions of the Ramsey problems of the Lagrangians of the optimal policy problem of the domestic and foreign policymaker are then given by:

$$L^{CMN} = \dots + U(C_{t}, L_{t}) + \beta U(C_{t+1}, L_{t+1})$$

$$+ \frac{1}{\beta} \Lambda^{CMN}_{t-1} RC^{CM}_{t-1} + \Lambda^{CMN}_{t} RC^{CM}_{t} + \beta \Lambda^{CMN}_{t+1} RC^{CM}_{t+1}$$

$$L^{*CMN} = \dots + U(C^{*}_{t}, L^{*}_{t}) + \beta U(C^{*}_{t+1}, L^{*}_{t+1})$$

$$+ \frac{1}{\beta} \Lambda^{*CMN}_{t-1} RC^{CM}_{t-1} + \Lambda^{*CMN}_{t} RC^{CM}_{t} + \beta \Lambda^{*CMN}_{t+1} RC^{CM}_{t+1}$$
(38)
(38)
(38)

Taking derivatives of L^{CMN} with respect to Λ_t^{CMN} , and $C_t, C_t^*, L_t, L_t^*, \pi_{H,t}, \pi_{F,t}^*, TOT_t$, and R_t summarizes the optimal policy of the domestic monetary authority, conditional on a foreign policy R_t^* . Equivalently, the derivatives of L^{*CMN} with respect to Λ_t^{*CMN} , and $C_t, C_t^*, L_t, L_t^*, \pi_{H,t}, \pi_{F,t}^*, TOT_t$, and R_t^* summarize the optimal policy of the foreign monetary authority, conditional on domestic policy R_t . The full Nash equilibrium can then be obtained by combining the Ramsey optimality conditions of the domestic and foreign policymakers (it should be noted, that the first order condition w.r.t. the vector of Lagrange multipliers of both policymakers are identical, that is, they simply return the equations of the competitive equilibrium).

Under financial autarky (incomplete markets bond economy) the uncoordinated Ramsey problem can similarly be defined, replacing RC_t^{CM} with the relevant vector of Ramsey constraints under financial autarky (bond economy), RC_t^{FA} (RC_t^{IM}), and attaching the corresponding Lagrange multipliers instead of Λ_t^{CMN} and Λ_t^{*CMN} : in the financial autarky world these are denoted by Λ_t^{FAN} and Λ_t^{*FAN} , in the incomplete markets bond economy by Λ_t^{IMN} and Λ_t^{*IMN} . As in the case of coordination, the incomplete markets bond economy consists of two financial market related equilibrium conditions, and the vectors of Lagrange multipliers

¹²This is in contrast to Faia and Monacelli (2004) and Liu and Pappa (2005), who take the each country's own optimality conditions as a constraints in the uncoordinated Ramsey problem (taking as given the other country's consumption, labor and inflation choices in addition to the policy instrument).

are correspondingly expanded to $\Lambda_t^{IMN} = [\lambda_{H1,t}^{IMN}, \lambda_{F1,t}^{IMN}, \lambda_{H2,t}^{IMN}, \lambda_{F2,t}^{IMN}, \lambda_{H3,t}^{IMN}, \lambda_{F3,t}^{IMN}, \lambda_{4,t}^{IMN}, \lambda_{5,t}^{IMN}]$ and $\Lambda_t^{*IMN} = [\lambda_{H1,t}^{*IMN}, \lambda_{F1,t}^{*IMN}, \lambda_{H2,t}^{*IMN}, \lambda_{F2,t}^{*IMN}, \lambda_{H3,t}^{*IMN}, \lambda_{F3,t}^{*IMN}, \lambda_{4,t}^{*IMN}, \lambda_{5,t}^{*IMN}].$

2.4 Parameterization

The parameterization of the model is summarized in Table 1. The discount factor β is taken to be 0.99, implying an annual interest rate of about 4 percent. Parameter θ is taken to be 6, which implies a markup over marginal cost of about 20 percent. Parameter γ (γ^*), which is the weight on domestic good in the domestic (foreign) consumption basket, is set to 0.75 (0.25) in the baseline case, implying that there is positive home bias. Following Bergin et al. (2007) and Faia and Monacelli (2004) the parameter of the quadratic adjustment cost in price setting is taken to be 50. The degree of risk aversion, σ , is considered to be 1 in the baseline parameterization (which implies log utility in consumption), the inverse Frisch elasticity of labor supply, κ , is equal to 3, a value commonly used in the real business cycle literature. The production subsidy parameter is set such that it offsets the monopolistic competition distortion, that is, τ , is set equal to $1/(\theta - 1)$. As for the exogenous processes, I consider a technology shock persistence of ρ_Z , $\rho_Z^* = 0.95$ and standard deviation of the shock of σ_Z , $\sigma_Z^* = 0.01$.

Finally, I consider a wide range for the value of the trade elasticity, ranging from goods being very complementary in consumption to goods being very substituable. There is no consensus on the choice of the value of this elasticity in the literature. In the trade literature, Trefler and Lai (1999) estimate, for individual goods, very high trade elasticities ranging between 1.2 and 21.4. In the (real) business cycle literature, the trade elasticity is typically taken to be lower. Backus, Kehoe and Kydland (1995) use elasticities between 0 and 5, Chari et al. (2002) assume a value of 1.5, Anderson and van Wincoop choose values between 5 and 10. A number of recent contributions have also emphasized the role of a low elasticity of intratemporal substitution (well into the complementarity region) together with an incomplete financial markets structure in the transmission of productivity shocks across countries, in particular in addressing stylized facts on international relative prices such as exchange rate volatility, terms of trade volatility or to replicate the empirical stylized fact of an appreciating TOT in response to a productivity increase (see e.g., Heathcote and Perri (2002), Corsetti, Dedola and Leduc (2008), Thoenissen (2008), Enders and Mueller (2006)). As I show, the value if the trade elasticity is a most crucial parameter in determining the influence of terms of trade in considerations of optimal monetary policy in an open economy.

3 Results

3.1 Ramsey Steady State

Section 2 has shown that the Ramsey equilibrium under the various financial market assumption and under coordination or Nash is obtained as the system of equations of first order conditions derived from the appropriate Ramsey problem. To determine the long-run inflation rate associated to the optimal policy problems above, one needs to solve the steady-state versions of the set of efficiency conditions. In all economies and regimes considered, the steady state (gross) inflation rate associated the to optimal policy problem is found to be equal to 1, as can be seen from the first order condition with respect to $\pi_{H,t}$ and $\pi_{F,t}^*$.¹³ Hence the Ramsey planner would like to generate an average (net) inflation rate of zero. The intuition for this result is simple. Under commitment, the planner cannot systematically affect the economy through monetary surprises (that would be aimed at eliminating the inefficiency related to market power in the goods market) and, thus, there is not a terms of trade externality as the one discussed in Corsetti and Pesenti (2001) and Clarida et al. (2002). That is, the planner cannot on average resort to movements in inflation to alter the relative purchasing power of its residents. The planner aims at choosing a long-run inflation rate that minimizes the cost of adjusting prices, which is summarized by the quadratic term. The openness dimension of the desire of adjusting the terms of trade can, therefore, drive the planner's behavior only in the presence of equilibrium fluctuations (as induced by country-specific shocks) around the same long-run steady state.

3.2 Transmission under flexible versus sticky prices

To facilitate the analysis of optimal monetary policy, I first examine a useful benchmark in which price adjustment is flexible, and then describe the dynamics under sticky prices. Under flexible prices a productivity increase in the domestic economy leads to a higher abundance of domestic goods. This translates into a decrease in the price of domestic goods resulting in a depreciation of the domestic terms of trade, making domestic goods relatively cheaper and channeling world demand towards domestic goods. Figure 1 shows the responses to the domestic productivity shock of major variables for the three financial market structures (CM, FA, and IM-Bond) and for the case where goods are either substitutes ($\omega = 3$), complements ($\omega = 0.7$) or are unit-elastic ($\omega = 1$).

Let's focus first on the case of goods being *substitutes* and consider the scenario of complete financial markets. The increase in domestic productivity leads to a domestic consumption increase, labor effort rise as the home economy gets more productive and the terms of trade deteriorate. Enjoying a more favorable price and because it is easy to sustitute to the now more abundant domestic good the foreign country also benefits from the domestic productivity shock. In particular, under complete markets, the terms of trade depreciate just enough to equalize the marginal utility benefit from the productivity shock in both countries, as dictated by the risk sharing equation. In the other extreme case of financial autarky, the response of the terms of trade is somewhat less pronounced. While the terms of trade still depreciates as an equilibrium response to the now more abundant domestic goods, it does so to a much lesser extent than where the marginal utility gain in the foreign economy is as high as in the home country. As no state-contingent assets have been traded promising the Foreign country

$$0 = \xi_{2H,t}^{i} \alpha \left(\pi_{H,t} - 1 \right) + \begin{bmatrix} \left(\xi_{3H,t}^{i} - \xi_{3H,t-1}^{i} \left(\frac{C_{t}}{C_{t-1}} \right)^{-\sigma} \frac{f^{p_{H}}(TOT_{t})}{f^{p_{H}}(TOT_{t-1})} \right) \\ \left[\alpha \left(2\pi_{H,t} - 1 \right) \right] - \xi_{1H,t-1}^{i} R_{t-1} \left(\frac{C_{t}}{C_{t-1}} \right)^{-\sigma} \frac{f^{p_{H}}(TOT_{t})}{f^{p_{H}}(TOT_{t-1})} \end{bmatrix}$$

which, at steady state (as $\xi_{3H,t}^i = \xi_{3H,t-1}^i$ and $\xi_{1H,t}^i = 0$) implies

$$0 = \xi_{2H,t}^i \alpha \left(\pi_{H,t} - 1 \right).$$

¹³This is the case even if the monopolistic distortion were not offset. In particular, the Ramsey first order condition w.r.t. $\pi_{H,t}$ is given by:

part of the benefits, Home labor effort does not increase, the expansion in domestic output is therefore lower than in the complete markets case, and the fall in the price of domestic goods relative to foreign goods (that is, the terms of trade deterioration) in turn less pronounced. When goods are substitutes the TOT and labor move too little under incomplete financial markets relative to CM.

The transmission of the productivity shock is somewhat different when goods are *comple*ments. Generally speaking, a lower elasticity of substitution implies that for any given change in quantities, higher movements in the price are necessary to bring about these movements in quantities. That is, under all financial market structures, the terms of trade responses are now much stronger than in the case where goods are substitutes. In addition, the TOT now depreciates more in the case of incomplete financial markets than under complete markets. Because home and foreign goods are complementary in utility from consumption, the (productivity-induced) higher abundance of domestic goods also leads to a higher demand for foreign goods. If markets are complete the foreign country is therefore bound to expand its output by increasing its labor effort which tends to take some of the pressure of the terms of trade increasing. Under financial autarky such an increase in foreign output is absent, as a result the increased demand for the foreign goods without a counterbalancing increase in supply for it leads to a deterioration of the terms of trade that is even stronger. The lower the trade elasticity, the stronger is the terms of trade depreciation, and the foreign country increasingly benefits from the domestic productivity increase. Summarizing, now, when goods are complements the TOT move too much under incomplete markets relative to CM.

Finally, we turn to the case in which goods are *unit-elastic*. If the elasticity is unity then relative price changes are completely offset by changes in output volumes. In this knife-edge case, the income effect of the required terms of trade depreciation (given the relatively higher productivity in Home) balances the incentive to switch expenditure towards Home goods: relative wealth is always unaffected in response to country specific shocks and that complete risk sharing is always obtained independent of the financial market structure assumed.¹⁴

Under sticky prices, it is costly for firms to change their prices which as a result don't adjust instantaneously. As is well known in the literature, a policy of producer price targeting would, however, lead to an exact replication of the flexible price allocation. If producer prices P_H and P_F^* are rigid, the policymaker can initiate a nominal depreciation of the home currency (a higher ε) such that the home terms of trade worsens, such that its response matches the one under flexible prices. When the home currency weakens, Home goods are cheaper relative to Foreign goods in both Home and the Foreign country. As demand shifts in favor of the goods with the lowest relative price, world consumption of Home goods increases relative to consumption of Foreign goods, which is known as "expenditure switching" effect of the exchange rate. While the replication if the flex-price allocation is possible, the adjustment under sticky prices requires action on the part of the monetary policymaker. Also, more importantly, it is not obvious that this strategy is the one an optimizing policymakers will follow in an open economy. The next sections turns to the results on the effect of imperfect risk sharing on optimal monetary policy.

¹⁴Strictly speaking, the threshold where relative price changes are completely offset by changes in output volumes lies only at unity because of my assumption of log-utility (that is a coefficient of relative risk aversion, $\sigma = 1$). More generally, as shown by Benigno and Benigno (2003) this threshold depends on both the intraand the intertemporal elasticity of substitution, and lies at $\omega = \frac{1}{\sigma}$.

3.3 The Role of Financial Market Structure and the Trade Elasticity for Stabilization

Allowing for a non-unitary elasticity implies that terms-of-trade volatility becomes important in the consideration for optimal policy, which many previous contributions to the literature have not addressed. This section presents results on how the structure of international asset markets can change the way monetary policy should be conducted and analyzes the implications of the terms of trade considerations. The desire of adjusting the terms of trade (or the real exchange rate) is generally sufficient to induce the planner to deviate from choosing a constant markup allocation. However, as outlined in section 3.1 these considerations can drive the planner's behavior only in the presence of equilibrium fluctuations (coming from country-specific shocks) around the long-run steady state. Therefore, the "optimal policy" is studied here in the sense of optimal stabilization in response to shocks. The shock considered throughout this section is a 1% increase in domestic productivity.

Figure 2 studies the optimal producer price inflation responses on *impact* of a the productivity shock, that is, unlike the regular impulse responses it ignores the time dimension of the shock. The left panel of the figure displays the impact responses on a relatively small scale, which is useful to understand the pattern of inflation responses around the treshold of the unit-elasticity. The same figure is then reproduced a second time, plotted on a larger scale, which turns out to be useful when studying the responses under incomplete markets and low elasticities. Figure 3 displays the impact behavior of the nominal interest rate. The optimal impact responses of domestic and foreign producer price inflation are depicted over a large range of the elasticity of substitution between domestic and foreign goods (ranging from very complementary goods to very substitutable goods), and for the various scenarios of financial markets. A central result, which becomes immediately aparent upon inspecting Figure 2 is that, for all cases but the one of perfect risk sharing and coordination, the implications are that deviations from full (producer) price stability are optimal. While, independent of the financial market assumption, a policy of keeping producer price inflation at zero would replicate the flexible price outcome, this is found to be the optimal policy only in the case of complete markets and coordination, or in the special case of a unit elasticity and therefore automatic full risk sharing.

To better understand why this is the case we would like to also study the responses of other variables of interest. Figures 4 and 5 therefore display the behavior of the terms of trade, the consumption and labor responses in the domestic economy under the various scenarios, by looking at differences of the responses of these variables to the responses that would occur in a flexible price version.

As discussed previously, in response to a 1% productivity increase in the domestic economy, the terms of trade depreciate, channeling demand to the now more abundant domestic good, both under flexible prices or the sticky price optimal monetary policy economy. Figure 4 shows that in the case of complete financial markets under policy coordination the difference of the TOT response under optimal policy from the response under flexible prices is zero at all values of the trade elasticity. The TOT under CM and coordination responds *exactly* as in the flexible price world. Also, under the special case of a unit-elasticity, financial market structure becomes unimportant as perfect risk sharing is automatically obtained over compensating movements in the international relative price. For all other cases the responses of the sticky-price optimal and the flexible-price models differ: we observe that the terms of trade either appreciate (CM, Nash) or depreciate (FA, Nash and Cooperation) relative to the flexible price responses, in line with the observation that pure producer price inflation targeting is not found to be optimal.

In particular, the mechanism behind these patterns is as follows: under *complete markets*, when risk sharing is perfect, it always optimal to to replicate the flexible price allocation as (as seen by the firm black line from Figure 2) price stickiness is the only distortion in the economy and the world policymaker maximizes world welfare. However, when acting uncoordinated, the policymaker of each country fails to take into account the effect of his policy choice on the other country's welfare. As a result the home authority finds it optimal to follow a policy in which the TOT vary somewhat less than under flexible prices if goods are substitutes: in response to the productivity increase the TOT is less depreciated relative to a flexible price outcome and producer price inflation is negative on impact. As consumption risk is shared and domestic goods can easily be substituted by foreign goods, the less pronounced TOTresponse aims at having to increase employment by a little less, which would be increasing their welfare, as this is done with the prospect of keeping the same utility from consumption. In a Nash equilibrium, however, this attempt is unsuccessful, as *both* policymakers have the incentive to let the terms of trade (or the real exchange rate) fluctuate less that what would be dictated by perfect risk sharing. As a result the TOT do not move 'enough', and while the uncoordinated planner succeeds in generating a lower volatility of labor effort, consumption volatility increases, which worsens overall welfare.

When goods are complements, the incentive for the home policymaker to contract the employment response and push some of the work effort to the foreign economy is absent, as foreign goods consumption cannot substitute consumption of domestic goods. On the contrary, the incentive is to render foreign goods even cheaper. As a result, when goods are complements, producer price inflation is positive following the domestic productivity increase, and the TOT is more depreciated relative to its flexible price response. Only in the case of a unit elasticity of intratemporal substitution the economies are insular with respect to TOT movements and the Nash outcome and coordination deliver the same result (of a prescription of price stability as optimal policy).

When we consider the scenario of *financial autarky*, the Ramsey planner now has to take another distortion into consideration in the design of its optimal policy as, in addition to nominal rigidities, the world is now one with a too low degree of international risk sharing. Figure 2 shows that the TOT is found to be more depreciated (compared to a flexible price scenario) when goods are substitutes, and the inflation response is positive. If a non-coordinated policymakers now were to reduce employment, this would still benefit agents by increasing the utility of leisure; unlike under complete markets, consumption risk is not shared and consumption is much more closely tied to current output. As productivity is currently high it pays off to increase output so much that the terms of trade depreciate even more than in the flexible price scenario. Under substitutes letting the TOT fluctuate more relative to the flexible price benchmark improves the risk sharing und pushes the TOT response somewhat closer towards how it would respond in a complete markets world. The higher TOT volatility therefore translates into a lower consumption volatility which improves welfare relative to flexible prices.

Only in the case where goods are complements domestic agents have an incentive to let the their terms of trade depreciate somewhat less (appreciate relative to a flex price world) and to contract output relative to the flexible price outcome. As a result, the prescription of an optimal policy flips again when crossing the area from goods being substitutes into

the complementarity region: in the latter case the TOT is found to be more appreciated relative to the flexible price response and producer price inflation decreases in response to a productivity shock. It is interesting to note, that under financial autarky the optimal policy of both the coordinated and uncoordinated policymakers have the same qualitatively implications of deviating from price stability. While even an uncoordinated planner finds it optimal to 'do some risk sharing', a coordinated policymaker will find it optimal to depreciate the terms of trade even more when goods are complements, taking account of the fact that production should take place in the more productive economy and that over a lower price of domestic goods both economies benefit (similarly, the coordinated planner will find it optimal to appreciate the terms of trade even more relative to the flex price case when goods are substitutes). From studying flexible price impulse responses in section 3.2 under the various financial market scenarios, we have seen that the TOT under incomplete markets (financial autarky or bond economy) depreciate too little (relative to the efficient economy with risk sharing) when goods are substitutes, but depreciate too much when goods are complements. A planner that, because of the presence of price rigidities, has some control over the terms of trade (or the real exchange rate), will therefore find it optimal to push it closer to the responses that would prevail in the complete markets case, thereby obtaining some risk sharing through the relative price. Policymakers under a sticky-price incomplete financial markets can therefore improve over the flex-price (but incomplete markets) outcome.

Finally, in the *incomplete markets-bond economy* case, the optimal responses to a domestic productivity shock lie somewhere in between the cases of complete markets and financial autarky. This finding is not surprising, considering that the availability of the international bond allows for some consumption smoothing. In turn, how easily the bond can be used in consumption smoothing depend crucially on the parameter of the portfolio adjustment cost, ψ . As ψ becomes very large, the policy prescriptions will closely follow the ones under financial autarky, if ψ is very small the optimal policy in the bond economy will be closer to the complete markets case. With the chosen value, it turns out that a policymaker under Nash competition follows a policy that is closer to the full risk sharing case, while a coordinated policymaker's policy matches closer that under financial autarky.

3.4 The Role Risk Sharing for Gains from Coordination and Gains over the Flexible Price Allocation

The fact that the policy prescription under Nash competition generally differs from the policy prescription under coordination implies that there are welfare gains from coordination. The welfare measures computed are conditional welfare, measured in terms of consumption equivalents. Table 2 shows that these are found increasing for elasticities of substitution away from unity and typically by an order of magnitude larger in the case of complementarity between domestic and foreign goods. Figure 6 also gives a graphical interpretation of these results. In the case where domestic and foreign goods can easily be substituted welfare gains from coordination are typically very small, and comparing complete markets with financial autarky typically larger under complete markets. The case is different when domestic and foreign goods are complementary in consumption. When goods are complements and the elasticity of substitution is very low, wealth effects from the movements in the relative price become very large under financial autarky. As a result, a coordinated planner, taking into account the relative price distortion, can acchieve much larger welfare gains. It is interesting to see

that welfare gains start to really increase drastically, as soon regions of the trade elasticity are reached that come closer to the asymptotic threshold where the sign of terms of trade switches in response to a productivity shock. This threshold lies at $1 - 1/2\gamma$ and was first discussed by Corsetti et al. (2008). It is of particular importance to include this region in a study of the effects of the degree of international risk sharing on optimal monetary policy, as this is the region of the trade elasticity which is able to reconcile the empirically observed lack of risk sharing with the model. The conclusion is therefore, that when risk sharing is low, gains from a international coordination of policies could be substantial.

Finally, the finding that even a coordinated policymaker finds it optimal *not* to replicate the (non-distorted) flexible price equilibrium means that there are welfare gains over the flexible price allocation whenever risk sharing is incomplete. A planner under incomplete markets (financial autarky or the bond economy) can improve upon the flexible price allocation by pushing the real exchange towards the case of perfect risk sharing. Table 3 computes the welfare gains of the various financial market regimes over the flexible price allocation, to give an indication of the quantitative importance of this finding. Figure 7 provides again a graphical interpretation. Clearly, under complete markets Nash competition leads to welfare losses over a flexible price (efficient) allocation. Interestingly, under financial autarky even Nash policymakers, even though they choose an inefficient level of the terms of trade volatility, are able to achieve welfare gains over a flexible price allocation.

4 Conclusion

The analysis of this paper has shown that the elasticity of intratemporal substitution and assumptions on the international financial market structure are important determinants of optimal monetary policy in the open economy. In particular, a purely inward-looking policy of producer price stability is found to be optimal only in the very special case in which financial markets are complete and policymakers act coordinately, or in the case of a unit trade elasticity which provides automatic perfect risk sharing. In all other cases it is optimal for monetary policymaker to not only consider stabilizing internal prices but to consider also the variability of international prices as the terms of trade (or the real exchange rate) in shaping their policy. In all but the special case of the unit elasticity, there are gains from policy coordination to be achieved, which become quite substantial when the trade elasticity is low and financial markets are incomplete.

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6 Tables

Table 1: Model Parameters

discount factor	β	0.99
elasticity between varieties	θ	6
home bias	$\gamma, 1-\gamma^*$	0.75
persistence of productivity shock	$ ho_Z, ho_Z^*$	0.95
standard deviation of productivity shock	σ_Z, σ_Z^*	0.01
coefficient of relative risk aversion	σ –	1
Rotemberg price adjustment cost parameter	α	50
Production subsidy offsetting monopolistic competition distortion	au	$1/\left(heta -1 ight)$
Portfolio adjustment cost parameter	ψ	0.00074
trade elasticity between H and F consumption goods	ω	$\epsilon \left[0.7, 3 ight]$

Table 2: Welfare gains from monetary policy coordination

ω	0.15	0.25	0.45	0.55	0.65	0.85	1	1.5	2	3	5
	$1e-02^{*}$	1e-02*	1e-02*	$1e-02^{*}$	$1e-02^{*}$	1e-02*		1e-02*	$1e-02^{*}$	$1e-02^{*}$	$1e-02^{*}$
CM	0.0119	0.0147	0.0036	0.0009	0.0004	0.0000	0	0.0001	0.0001	0.0000	0.0001
FA	0.4737	7.8751	0.0776	0.0126	0.0005	0.0001	0	0.0001	0.0001	0.0000	0.0000
IM	0.4678	15.351	1.0828	0.0828	0.0058	0.0002	0	0.0002	0.0004	0.0004	0.0004

Table 3: Welfare gains from optimal policy over flexible price allocation

ω	0.15	0.25	0.45	0.55	0.65	0.85	1	1.5	2	3	5
	1e-02*	1e-02*	1e-02*	1e-02*	1e-02*	1e-02*		1e-02*	1e-02*	1e-02*	1e-02*
CMC	0	0	0	0	0	0	0	0	0	0	0
FAC	1.1433	25.3753	2.0011	0.1966	0.0286	0.0008	0	0.0003	0.0003	0.0002	0.0001
IMC	0.9806	24.8341	1.5775	0.1296	0.0197	0.0006	0	0.0003	0.0003	0.0002	0.0001
CMN	-0.0119	-0.0147	-0.0036	-0.0009	-0.004	-0.0000	0	-0.0001	-0.0001	-0.0000	-0.0001
FAN	0.6695	17.4802	1.9241	0.1840	0.0281	0.0007	0	0.0003	0.0003	0.0002	0.0001
IMN	0.5127	9.4452	0.4946	0.0468	0.0139	0.0004	0	0.0001	-0.0000	-0.0002	-0.0003

7 Figures



Figure 1: Impulse responses to a domestic productivity shock under flexible prices



Figure 2: Impact responses of optimal domestic and foreign producer price inflation to a domestic 1 % productivity shock



Figure 3: Impact responses of optimal domestic and foreign nominal interest rates to a domestic 1 % productivity shock



Figure 4: Differences of optimal TOT impact responses over flexible price TOT impact responses (to a domestic 1 % productivity shock), depending on the trade elasticity



Figure 5: Differences of optimal consumption and labor impact responses over flexible price impact responses (to a domestic 1 % productivity shock), depending on the trade elasticity



Figure 6: Welfare gains from coordination



Figure 7: Welfare gains over policy of price stability