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Implications of Firm Heterogeneity for Offshoring, Wage and Employment

Xi Chen[1]

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Keywords: Heterogeneous firms, productivity, offshoring, fair wage, employment

JEL: D21, D24, F12, F16, F66

The author

1 STATEC Research (National Statistical Office of Luxembourg), 13, rue Erasme, L-1468 Luxembourg.
E-mail: xi.chen@statec.etat.lu

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Implications of Firm Heterogeneity for Offshoring, Wage and Employment

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1 Introduction

In a globalized economy, firms increasingly rely on their foreign production units, which allows them to produce at a lower cost, to raise their revenue and eventually leads to higher employment level. However, the effects of offshoring are highly contested (Scheve and Slaughter, 2001). Much of the public concern is due to the expectation that cheaper foreign inputs may replace tasks previously done by domestic labor, and cause displacement of workers and lower the social welfare (Feenstra and Hanson, 1995, 1997). The deteriorated labor market condition from the financial crisis of 2008 has exacerbated the public discontent with globalization and allowed the rise of populist political movements around the globe. In response, governments are trying to propose industrial policies that minimize the adverse effects of globalization. However, the good performance of these measures requires a deep understanding of the individual behavior and their sector implications. The difficulty here is the massive amount of heterogeneity within narrowly defined sectors. A given shock may help certain firms and harm others, which then lead to an ambiguous outcome at the sector-level.

In this paper, I consider heterogeneity in firms' productivity and examine its implications for the wage setting and the division of production tasks between domestic and foreign workers. Following Melitz (2003), I develop a general equilibrium model with monopolistic competition. The proposed model allows for labor market frictions, which is characterized by a fair wage condition (Egger and Kreickemeier, 2009). This rent-sharing mechanism implies that more successful firms pay higher wages. The model depicts an open economy where firms have access to the international labor markets and may replace the tasks previously done by their domestic workers with the cheaper foreign inputs (Groizard et al., 2014). The combination of these features results in a model where heterogeneous firm differs in their productivity, and both wages and offshoring level are functions of productivity.[1] This paper contributes to the literature in three aspects: the paper examines (i) the relationship between cost structure and productivity; (ii) the equilibrium implications of a production technology with fair wage and offshoring, and (iii) the impacts of production-side changes on employment and welfare.

In the literature of heterogeneous firms, most of the studies assume that more productive firms produce at a lower marginal cost. A typical example is Melitz (2003), in which firms' marginal cost is simply the inverse of their productivity. Egger and Kreickemeier (2009) adds a fair wage condition to the Melitz (2003) model,

[1]This setup belongs to a class of heterogeneous firm models in which productivity is not merely a Hicks-neutral technical term, but it can also affect many aspects of production organization.

that productivity also influences the marginal cost via the wage channel. This paper considers an additional offshoring channel through which productivity influences the marginal cost function. The study shows that the production technology of heterogeneous firms in an offshorable sector with rent-sharing motivated workers implies an intricate relationship between productivity and marginal cost . For instance, on the one hand, more productive firms have lower marginal cost for a given wage and offshoring level. On the other hand, more productive firms also pay the higher wage that leads to higher marginal cost, but at the same time, they also offshore more to reduce their marginal cost.

The second contribution can be summarized as the following. Individual firm reacts directly to a shock as well as indirectly to the equilibrium shift caused by the shock. The paper proposes a tractable framework to study how changes that alter the productivity-cost relationship, can influence the firm behavior and the sectoral equilibrium including, market entry cutoff, average productivity, profit, and revenue. The analysis points out that a shock can shift the sectoral equilibrium when its impacts on different firms within the sector differ. For instance, a higher average foreign wage can lead to a lower entry cutoff and a lower sector-average profit of active firms. This is because higher foreign wage increases the burden on these firms, which are highly productive and heavily relying on their foreign production units. The paper thus shows that firm heterogeneity is crucial for determining how shocks affect the sectoral equilibrium.

The third contribution is the analysis of employment and welfare responses to the changes in firms' wage setting and offshoring incentive. The paper first shows that, at the firm-level, a reduction of offshoring incentive (for example, a rise of foreign wage) may not necessarily increase the absolute number of domestic workers, but it will increase the relative share of domestic workers in the total labor force. The study then points out that, at the sector-level, the effects can differ significantly from those found at the firm-level. The main reason is that the aggregation of heterogeneous firms distorts the individual outcomes, where the distribution of heterogeneity determines the propagation of shocks from firms to the sector. Therefore, a shock that helps to increase the domestic employment of some firms may not be able to generate the domestic employment gain at the sector-level. Finally, this analysis shows that even if a shock is leading a positive effect on the domestic employment, it may have a negative impact on the utilitarian welfare. These findings point out that we need to understand the overall effects of changes regarding offshoring behaviours and not only focus on individual responses.

The paper is organized as the following. I begin in Section 2 with a brief literature review.[2] I then present in Section 3 the empirical evidence using the U.S multinational enterprises data. Here, the main question is how does the within-firm offshoring affect firms' domestic and foreign labor composition. The estimation results are inconsistent with the idea that firms substitute domestic labor with offshoring, and show significant variation among sectors. Thus, the empirical exercise motivates the following theoretical analysis to explorer the within-sector mechanisms that shape the employment-offshoring relationship. This section also reviews some recent firm-level evidence and discuss the theoretical implications for this paper. In Section 4, I describe the main features of a theoretical framework with an unspecific cost structure that characterizes firms' production technology. The generic functions simplify the exposition and show in a general way the equilibrium implications of production-side shocks, thus paving the way for the analysis under more specific functional forms. Section 5 introduces an offshoring model based on Groizard et al. (2014)'s approach where firms choose an optimal offshoring level by comparing the domestic and foreign wages. Section 6 describes the fair wage condition, under which both domestic and foreign wages increase with productivity. Using results from the general model, Section 7 performs a comparative statics analysis by focusing on changes in (i) the wage difference between domestic and foreign workers; (ii) the degree of fairness concern. The first part of Section 8 focuses on the firm-level question: how a reduction of offshoring incentives affects firm domestic labor demand? Then turning to the sector-level in the second part of Section 8, the analysis shows how shocks reverberate throughout the local labor market. Finally, Section 8 also discusses the welfare implications of changing wage setting using the per capita output as the utilitarian welfare measure. Section 9 provides the final remarks.

2 Related literature

The seminal paper of Melitz (2003) provides a highly tractable theoretical framework for modeling firms export decisions, in which heterogeneous firms face entry costs. The Melitz model is the main workhorse for dealing with various issues in international economics.[3] There are, however, two important questions that the Melitz-type model does not provide answers. First, Melitz (2003) assumes perfect competition in the labor market. All firms in his model face the same labor cost, and neither trade nor productivity affects workers real wage. The second missing

[2]Hummels et al. (2018) provides a comprehensive survey of literature on labor market implication of offshoring.

[3]Redding (2011) provides a survey of recent theoretical literature on heterogeneous firms and trade.

component of the Melitz model is pointed out by Bernard et al. (2007) that following (Krugman, 1979; Melitz, 2003), the new theories of heterogeneous firms were developed exclusively to explain export behavior and yield few predictions in other aspects of firms' international activities, such as importing or offshoring.

Egger and Kreickemeier (2009) addresses the first limitation by adding a fair wage condition to the Melitz (2003) model. In this framework, firms' performance is linked to workers' welfare, as more productive firms pay higher wages. Then they analyze the impact of export liberalization on wage inequality and unemployment under the fair wage consideration.[4] A similar model is Helpman et al. (2010), where heterogeneous firms face an uncertainty on *ex post* match-specific workers' ability shocks. In this framework, wages are the outcome of a bargaining process that depends on firm performance and screening ability. Thus, using the search and matching theory, Helpman et al. (2010) is also able to generate a within-industry wage variation for *ex ante* homogeneous workers. The two papers represent a growing body of literature that combines trade theory with various forms of labor frictions, which also includes Helpman and Itskhoki (2010); Amiti and Davis (2012); Egger and Kreickemeier (2012).

Following Egger and Kreickemeier (2009), the current heterogeneous firm model with fair wage consideration addresses only one aspect of globalization: exporting. However, the role of imports and offshoring for labor market outcomes has featured more prominently in both public and academic debate.[5] Biscourp and Kramarz (2007) find that there is a strong correlation between increased imports and job destruction in France, while exports associates positively with job creation. Mitra and Ranjan (2010) develops a theoretical model with perfect competitive markets that highlights the effects of offshoring on the domestic labor market. In the Mitra and Ranjan (2010) model, firms have an outside option, where the imported intermediate goods substitute the domestic labor. The outside option gives additional bargaining power to trading firms in the wage negotiation. Thus, the model predicts that the trade opening leads to a reduction of employment and a decline of wage in the sector that labor is offshorable. Groizard et al. (2014) proposes a heterogeneous firm model with monopolistic competition and analyzes the employment implications of a change in offshoring cost. They find a decline in offshoring costs affects labor reallocation both within a firm and between them. The two key parameters determining this effect are the elasticity of substitution between domestic and foreign labor, and

[4]The earlier studies on the implications of the fair wage also include Akerlof and Yellen (1990); Fehr and Gächter (2000).

[5]In a general sense, there are few conceptual differences between importing foreign inputs and relocating domestic jobs abroad.

the elasticity of substitution between varieties of differentiated goods. The current paper inherits most of its features from Groizard et al. (2014)'s offshoring model, and combines them with Egger and Kreickemeier (2009)'s fair wage consideration. The current paper differs from the two papers in the following aspects. First, the paper extends the analysis in Egger and Kreickemeier (2009) by focusing on the labor market effects of offshoring and the substitution between domestic and foreign workers. Second, it extends Groizard et al. (2014) in the sense that, due to the fair wage consideration, heterogeneous firms in the current model face different incentives to offshore and have different offshoring intensity. This feature is crucial to explain the discrepancies between individual and sector effects of a production-side shock on the domestic employment.

Grossman and Helpman (2007) and Mitra and Ranjan (2013) also study the impact of workers' preference for fairness on firms' organization of production in an open economy where the domestic tasks are offshorable. However, the current paper differs from these papers in two important aspects. First, both Grossman and Helpman (2007) and Mitra and Ranjan (2013) focus on understanding how fairness and substitution between skill and unskilled co-workers can influence firms' behavior and the labor market outcomes. In a different setting, I examine the relationship between domestic and foreign worker and its implications for the domestic labor market. Second, the two papers assume the perfect competition in the product market and homogeneous firms, while my paper is set in the monopolistic competition framework with heterogeneous firms. The two other closely related papers are Sethupathy (2013) and Egger et al. (2016), which also set the analysis in a framework of monopolistic competition and heterogeneous firms. Sethupathy (2013) examines the wage and the firm-level employment effects of offshoring. This paper provides theoretical and empirical evidence that domestic wages rise at firms, which take advantage of offshoring opportunity, and domestic job loss is unlikely due to the expansion of offshoring activities. Egger et al. (2016) investigate the role of offshoring in the phenomenon of job polarization. They argue that the polarization is resulting from a reallocation of labor across heterogeneous firms and can be explained by the increased production offshoring from high- to low-wage countries. The current paper differs from the two in the objectives of analysis. First, as both offshoring and wage are endogenous, the present article does not try to establish the causal relationship, but it analyses the effects of exogenous changes in wage setting given firms' offshoring options. Moreover, the present article focuses on sectoral employment rather than individual firms' labor demand.

3 Empirical evidence

This section presents some descriptive and reduced-form evidence on the relationship between labor demand and offshoring. The data used in the analysis are constructed from the U.S. Bureau of Economic Analysis (BEA)'s multinational enterprises (MNE) database. Slaughter (2000) provides a detailed account of this database. Here, I only summarize the information needed for the offshoring analysis. The BEA collects data on the activities of U.S. multinational enterprises together with their foreign affiliates, and provides statistics for 32 manufacturing sectors over a period from 1999 to 2015.[6] Before presenting the empirical strategy, I first define the key variables of this analysis and introduce the main questions.

In this analysis, I consider all U.S. headquartered parent companies (PC) in the manufacturing sector and focus only on their majority-owned foreign affiliates (MOFA). [7] The goal is to examine how MNE's international activities affect the domestic employment at the sector-level. I distinguish two types of international activities, the *within-firm offshoring* and the *MNE transfer*. The within-firm offshoring refers to the intermediate inputs produced in the MOFA sent back to the U.S. PC for further processing. Implicitly, I assume that the MOFA situates in the upstream of production chain compared to the PC. I use the ratio between the imports of goods shipped to U.S. parents by foreign affiliates and the value-added of U.S. parents as a proxy for measuring the intensity of within-firm offshoring. [8] The MNE transfer refers to the transfer of the production stages from the U.S. parent firms to their foreign affiliates (Slaughter, 2000). The MNE transfer differs from the within-firm offshoring in the destination and use of foreign production units. The MNE transfer does not require the foreign outputs to be sent back to U.S. parents, but it accounts all outputs either sold in the host market or exported to some third markets. The corresponding empirical measures include the MOFA-PC ratio of value-added or fixed assets. In this analysis, the MNE transfer is not the primary variable of interest but is instead used as the control variable. The dependent variable is the domestic manufacturing employment, which is measured as the total number of workers in the U.S. manufacturing PC or the PC-MOFA labor ratio. The following subsection summarizes the evolution of these variables over the sample period.

[6]Note that four additional sectors at more aggregate-level are also reported.

[7]The MOFA are those that PC hold at least a 50% ownership status.

[8]Note that BEA-MNE database also includes the imports of goods shipped to U.S. parents by foreigners other than foreign affiliates, which can capture the arm's-length offshoring.

3.1 Descriptive statistics

Table 1: **Average annual growth rates of key variables (in %)** [a]

		1999-2007	2008-2015	1999-2015
Employment	L^{pc}	-2.72	0.76	-0.98
	$\frac{L^{pc}}{L^{mofa}}$	-4.52	-1.20	-2.86
Offshoring	$\frac{IM}{VA^{pc}}$	5.79	0.11	2.95
MNE transfer	$\frac{VA^{mofa}}{VA^{pc}}$	7.02	-2.92	2.05
	$\frac{K^{mofa}}{K^{pc}}$	12.79	1.12	6.95
Wage difference	$\frac{W^{mofa}}{W^{pc}}$	-0.79	-2.13	-1.46

[a]Source: BEA-MNE database. Note: the annual growth rate is calculated as the difference between current and past values divided by the past value. The superscripts pc and $mofa$ indicate type of entities; L^{pc} and L^{mofa} denotes sector-level employment; IM denotes the imports of goods shipped to U.S. parents by foreign affiliates; VA, K and W denote value-added, capital stock and wage, respectively.

The number of manufacturing workers in the PC fell with an average annual rate of -0.98% over the period from 1999 to 2015. Given an increasing employment in the MOFA, the decline of PC-MOFA labor ratio is even more dramatic with an annual rate of -2.86%. This decline seems to follow the trend of MNE transfer, where the growth of foreign productions is faster than their U.S. domestic counterparts (2.05%) with a significant pace of capital stock transfer (6.95%). The within-firm offshoring is also gaining with a rate of 2.95% per year. A possible explanation for the fell of domestic employment and the raise for the MNE transfer and the within-firm offshoring is the increasing domestic-foreign wage difference, where the U.S. became a relatively more expensive production location.

Note that dividing the sample period into two, different patterns emerged. In the first period, 1999-20007, the decline of domestic manufacturing employment, the rise of MNE transfer and offshoring are particularly pronounced. In the second period, 2008-2015, the U.S. manufacturing sector was gaining domestic employment. At the same time, the MNE transfer contracted regarding value-added ratio, and the within-firm offshoring progressed at a much slower pace. These observations seem to indicate some correlation between the domestic employment, the MNE transfer, and the within-firm offshoring. In the following regression exercises, I try to answer two questions. How does the within-firm offshoring affect the domestic employment controlling for the size of sectors, the wage effects, and the MNE transfer? Is the domestic-foreign wage differences influence the intensity of within-firm offshoring controlling for the MNE transfer?

3.2 Regression analysis

In the first regression analysis, I regress the PC's domestic labor demand on the within-firm offshoring intensity controlling for size, wage and MNE transfers:

$$\begin{aligned}\log L_{it}^{pc} &= \alpha_i + \alpha_t + \beta_1 \frac{IM_{it}}{VA_{it}^{pc}} + \beta_2 \frac{VA_{it}^{mofa}}{VA_{it}^{pc}} + \beta_3 \frac{K_{it}^{mofa}}{K_{it}^{pc}} \\ &\quad + \beta_4 \log VA_{it}^{pc} + \beta_5 \log K_{it}^{pc} + \beta_6 \log W_{it}^{pc} + \varepsilon_{it},\end{aligned}$$

where i indexes sectors; t indexes time; α_i denotes sector dummies; α_t denotes year dummies and ε is the error term. In the second model, the depend variable is the PC-MOFA labor ratio:

$$\begin{aligned}\log \frac{L_{it}^{pc}}{L_{it}^{mofa}} &= \alpha_i + \alpha_t + \beta_1 \frac{IM_{it}}{VA_{it}^{pc}} + \beta_2 \frac{VA_{it}^{mofa}}{VA_{it}^{pc}} + \beta_3 \frac{K_{it}^{mofa}}{K_{it}^{pc}} \\ &\quad + \beta_4 \log VA_{it}^{pc} + \beta_5 \log K_{it}^{pc} + \beta_6 \log W_{it}^{pc} \\ &\quad + \beta_7 \log VA_{it}^{mofa} + \beta_8 \log K_{it}^{mofa} + \beta_9 \log W_{it}^{mofa} + \varepsilon_{it}.\end{aligned}$$

The results are summarized in Table 2.

The results of the model (1) and (2) suggest that the within-firm offshoring affects the absolute number of workers in PC positively, and negatively the PC-MOFA labor ratio. The results in (3) and (4) by adding the sector and year dummies show that the effect of within-firm offshoring on the absolute number remains, while the effect on the PC-MOFA ratio vanishes. Thus, on the one hand, the within-firm offshoring seems to be positively correlated with the domestic employment. This may be because both domestic and foreign productions are simultaneously adjusted in response to the demand, and the imported MOFA intermediates and the domestic workers are complements. On the other hand, the result of the PC-MOFA labor ratio seems to be inconsistent with the idea that affiliate activities necessarily substitutes for PC activities. It suggests that in some sectors and years, the offshoring activities may affect the labor ratio negatively, but in other circumstances, the opposite holds. Thus, the empirical evidence here seems to indicate that some within-sector mechanisms may regulate the effect of within-firm offshoring on the domestic-foreign labor ratio. The sectoral variation is likely due to the difference in

Table 2: **Domestic employment and within-firm offshoring** [a]

	PC's employement (1)	PC-MOFA ratio (2)	PC's employement (3)	PC-MOFA ratio (4)
Intercept	**0.767** (4.518)	-0.058 (-0.124)	**-0.996** (-2.737)	**-1.877** (-3.940)
$\frac{IM}{VA^{pc}}$	**0.396** (3.795)	**-0.612** (-6.742)	**0.415** (3.943)	0.125 (0.909)
$\frac{VA^{mofa}}{VA^{pc}}$	**-0.605** (-9.009)	**1.410** (4.826)	**0.210** (3.547)	0.186 (0.903)
$\frac{K^{mofa}}{K^{pc}}$	0.0115 (0.402)	-0.145 (-2.270)	0.002 (0.081)	**0.150** (3.341)
$\log VA^{pc}$	**0.641** (11.317)	**0.923** (5.168)	**0.558** (11.604)	**0.331** (2.776)
$\log K^{pc}$	**0.290** (5.348)	0.1470 (1.694)	**0.200** (4.554)	**0.359** (5.290)
$\log W^{pc}$	**-1.190** (-25.326)	**-0.362** (-5.413)	**-0.393** (-5.412)	-0.167 (-1.834)
$\log VA^{mofa}$		**-1.092** (-5.102)		-0.335 (-2.080)
$\log K^{mofa}$		0.034 (0.375)		**-0.235** (-3.575)
$\log W^{mofa}$		0.091 (1.787)		**0.306** (7.931)
Sector Dummies	no	no	yes	yes
Year Dummies	no	no	yes	yes
R^2	0.951	0.707	0.993	0.945
Observations	541	541	541	541

[a]Source: BEA-MNE database. Note: The standard errors are estimated with heteroskedasticity-consistent estimator. The t values are reported in the parenthesis. The estimates that have a p value less than 1% are highlighted in bold print.

the production technology, the market structure, and the distribution of heterogeneous firms that make up the sector. In order to better elucidate the relationship and also to be able to interpret the data in a structural manner, I turn in the next section to develop a theoretical model.

The second piece of empirical evidence that will influence the theoretical analysis is the relationship between the intensity of within-firm offshoring and the domestic and foreign wage difference. Table 3 summarizes the result of regressing the intensity of within-firm offshoring ($\frac{IM}{VA^{pc}}$) on the wage difference ($\frac{Wmofa}{W^{pc}}$) by controlling for the MNE transfer ($\frac{VA^{mofa}}{VA^{pc}}$, $\frac{K^{mofa}}{K^{pc}}$ and $\frac{L^{mofa}}{L^{pc}}$). The estimation suggests that at the sector-level the domestic-foreign wage difference seems to motivate the within-firm offshoring. Thus, one objective is to use the model to show how a change in the foreign-domestic wage difference can affect the domestic employment given firms' offshoring options.

Table 3: **Within-firm offshoring and domestic-foreign wage difference** [a]

Intercept	**0.139**
	(4.416)
$\frac{Wmofa}{W^{pc}}$	**-0.039**
	(-4.441)
$\frac{VA^{mofa}}{VA^{pc}}$	**0.275**
	(7.410)
$\frac{K^{mofa}}{K^{pc}}$	-0.031
	(-1.938)
$\frac{L^{mofa}}{L^{pc}}$	**-0.116**
	(-2.625)
Sector Dummies	yes
Year Dummies	yes
R^2	0.8923

[a]Source: BEA-MNE database. Note: The standard errors are estimated with heteroskedasticity-consistent estimator. The t values are reported in the parenthesis. The estimates that have a p value less than 1% are highlighted in bold print.

3.3 Firm-level evidence and the modeling choices

In this paper, I focus on one possible explanation for the sectoral variation in the offshoring-employment relationship found in the previous empirical analysis: firm heterogeneity. Before turning to the theory, it is instructive to briefly review a series of firm-level studies that will guide the modeling choices in the next section.

Empirical studies of firms for a wide range of countries and sectors have documented substantial heterogeneity in their performance measures. For instance, Bernard et al. (2003) show that US manufacturing plants differ in their size, capital intensity, and productivity, even within narrowly defined sectors. These significant differences are strongly correlated with firm decision to engage in international transactions and wage bargaining.

Abowd et al. (1999) analyze a matched sample of one million French workers and a half million employing firms. They found that controlling for personal effects, high wage firms are more productive and more profitable. From the theoretical perspective, one way to explain this result is the theory of "gift exchange" advanced by Akerlof (1982). The key idea behind this theory is that workers have a preference for fairness and condition their effort on the offered wage. Workers provide a higher level of effort in exchange for a wage above some reference level that is considered as "fair". Profit-maximizing firms under the fair wage consideration may find it optimal to offer a wage that exceeds the market-clearing level. In the heterogeneous firm framework, the definition of fairness may vary depending on the firms' characteristics. Thus, this rent-sharing mechanism implies that ex-ante identical workers may face different compensation treatment in the equilibrium, with an offered wage correlated to certain aspects of their employers' characteristics. Abowd et al. (1999)'s finding relates rent-sharing to firms' performance and suggests that more successful firms set a higher reference compensation level and pay higher wages. This yields, in particular, a positive correlation between wage and productivity.

In most of the previous offshoring model, the main concern is firms' decision to whether integrate locally the production of intermediate inputs or offshore them–the extensive margin. In this paper, I focus on the intensive margin where all firms offshore, but differ in the level of offshoring depending on their payroll structure. The correlation between wage and productivity implies that more productive firms have the incentive to relocate a larger part of their production to foreign countries because they are facing a relatively higher domestic labor cost. Indeed, some empirical studies show more productive firms tend to be more active in offshoring (Tomiura, 2005, 2007). Thus, these empirical observations raise the concern that in an open economy with the higher correlation between wage and productivity, more productive firms may hire less domestic workers that implies a greater magnitude of job destruction due to offshoring.[9] This paper aims to show in a theoretical framework

[9]Note that this scenario, however, only describes one aspect of the productivity-employment nexus. Numerous studies reassure us by showing that more productive firms are generally larger regarding employment (Bernard et al., 2007). Clearly, in an open economy firms' productivity can be translated into employment via different conflicting channels. For instance, more productive firms are more likely to serve not only their domestic market but also the international markets, which generates higher employment to answer the higher demands (Melitz, 2003). In this paper, I limit the analysis to a sector where the only international activity is offshoring, exporting and

the different channels through which offshoring affects the domestic labor market. In the remaining sections, I first model the relationship between firms' offshoring, (domestic and foreign) wages, labor demands, and integrate it into a general equilibrium model with monopolistic competition. Then I focus on the question of how a change in the wages setting affects individual offshoring behavior and what are the implications for domestic employment.

4 The general framework

In this section, I start with a brief outline of demand and production with an unspecified cost structure; I then describe the equilibrium and derive the existence and uniqueness condition. Finally, I study the impact of a hypothetical production-side shock on the sectoral equilibrium.

4.1 Demand

The demand of final goods is characterized by the monopolistic competition (Dixit and Stiglitz, 1977). The consumer's preferences over a continuum of final good varieties is modeled by a CES function:

$$Q = \left[M^{\frac{1}{\sigma}} \int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where Ω is the measure of set that representing the mass of available varieties M. Q can be considered as an aggregate good with the aggregate price defined as

$$P = \left[M^{-1} \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}, \tag{2}$$

where $q(\omega)$ and $p(\omega)$ are the quantity and price of the individual variety; $\sigma > 1$ is the elasticity of substitution between any two varieties. This aggregate good is the numéraire with the normalized aggregate price $P = 1$. Thus, the aggregate expenditure R coincides with Q. The representative consumer maximizes her utility or consumption of aggregate good subjected to the budget constraint. Solving the consumer's problem yields the Marshallian demand equation of a given variety, ω:

$$q(\omega) = p(\omega)^{-\sigma} \frac{Q}{M}. \tag{3}$$

FDI are excluded.

4.2 Production

There is a continuum of firms, each producing a different variety ω. Before the production, each producer pays a sunk cost of entry, f_e and draws its productivity, which is distributed with the probability density function $g(\phi)$ and the cumulative distribution function $G(\phi)$. Productivity shock is the only source of heterogeneity in this model. After observing its productivity, a firm could choose to exit immediately because the market is not profitable, and there is also a constant rate δ that firm is forced to exit due to some natural disasters.

In this section, I do not restrict the discussion to a specific form of production technology. Instead, I consider a general cost structure that exhibits constant marginal cost with a fixed overhead cost and satisfies Assumption 1. This means that the fixed cost is incurred only once each production period and the marginal cost does not vary with the level of production. All firms face the same fixed cost denoted by f, but have a different level of marginal cost depending on their productivity, i.e., $c(\phi) > 0$ for all ϕ. This model covers a broad range of underlying production technologies, which can be multiple factors and may exhibit Hick-neutral or non-neutral technical change.

Assumption 1. *Firms with higher productivity produce at lower marginal cost, i.e., $\frac{\partial c(\phi)}{\partial \phi} < 0$, for all values of ϕ.*

The variable profit of a firm that produces one variety of good equals the revenues net of the associated costs. The profit maximization yields a pricing rule:

$$p(\phi) = \frac{\sigma}{\sigma - 1} c(\phi). \tag{4}$$

Based on (3) and (4), we can express firms' revenue and net profit as function of productivity:

$$r(\phi) = p(\phi)^{1-\sigma} \frac{Q}{M}; \tag{5}$$

$$\pi(\phi) = \frac{r(\phi)}{\sigma} - f. \tag{6}$$

4.3 Firm entry and exit

The static equilibrium of the general model is characterized by an entry threshold productivity level that only the firms have higher productivity than this value become active.[10] There is a large unbounded pool of potential entrants into the final good sector. Any firm has a productivity draw $\phi < \phi^*$ will immediately exit, where ϕ^* is the threshold value that identifies the least productivity level of active firms with the zero cutoff profit (ZCP) condition, $\pi(\phi^*) = 0$. For any firm has a productivity draw $\phi \geq \phi^*$, the net profit can be expressed as

$$\pi(\phi, \phi^*) = \left[\frac{c(\phi)}{c(\phi^*)}\right]^{(1-\sigma)} f - f. \tag{7}$$

Now, I define a weighted average of productivity level $\overline{\phi}$ in the way that $p(\overline{\phi}) = P = 1$. Given a threshold value, ϕ^*, this average productivity is determined implicitly by

$$\int_{\phi^*}^{\infty} \left[\frac{c(\phi)}{c(\overline{\phi})}\right]^{1-\sigma} g(\phi \mid \phi \geq \phi^*) d\phi = 1. \tag{8}$$

The direct implications of this definition are the following. The proofs are reported in Appendix A.1 and 2.

Lemma 1. *Given Assumption 1, the average productivity,* $\overline{\phi}(\phi^*)$ is an increasing function of the threshold value, ϕ^*.

Lemma 2. *The definition of* $\overline{\phi}$ *implies that,*

(i) *the average revenue defined as* $\overline{r} \equiv \int_{\phi^*}^{\infty} r(\phi) g(\phi \mid \phi \geq \phi^*) d\phi$, *equals the revenue of a* $\overline{\phi}$*−firm, i.e.,* $\overline{r} = r(\overline{\phi})$;

(ii) *the average profit defined as* $\overline{\pi} \equiv \int_{\phi^*}^{\infty} \pi(\phi) g(\phi \mid \phi \geq \phi^*) d\phi$, *equals the revenue of a* $\overline{\phi}$*−firm, i.e.,* $\overline{\pi} = \pi(\overline{\phi})$.

Equation (8) shows that the average productivity depends implicitly on the range of productivity levels within the sector, which is endogenously determined by ϕ^*− a self-selection process. Using the implicit function theorem, Lemma 1 indicates that how the endogenous self-selection process affects the average productivity. The choice of average defined in (8) also provides two convenient identities where the average revenue or profit within the sector equals the revenue or profit of an average productive firm (Lemma 2).

Since the mass of potential entrants is unbounded, the net value of entry should equal to zero to ensure an static equilibrium. This is the so-called free entry (FE) condition:

$$\overline{\pi} = \frac{\delta f_e}{1 - G(\phi^*)}. \tag{9}$$

[10] Note that in this paper, I limit the analysis to the case where firms draw their productivity from an exogenous distribution.

Using the ZCP condition and the FE condition, we can determine a pair of $\overline{\pi}$ and ϕ^* that characterizes the equilibrium. The existence and uniqueness of this equilibrium are guaranteed by the following condition. The proof is proved in Appendix A.3.

Proposition 1. *Given Assumption 1, the equilibrium characterized by (7), (8) and (9) exists and is unique.*

Note that, in the current model, the marginal cost of individual firms depends strictly on their productivity level. However, firms may also adjust their marginal cost in response to sector-wide variables, such as the unemployment rate and minimum wage. A more general specification of marginal cost function is $c[\phi, m(\overline{\pi})]$.[11] Under the more general specification, (7), (8) and the result in Proposition 1 remain unchanged when the following assumption is satisfied.

Assumption 2. *There is a functional separability in the marginal cost, such that* $c[\phi, m(\overline{\pi})] = \tilde{c}(\phi)m(\overline{\pi})$.

4.4 Impact of a production-side shock on the equilibrium

In this subsection, I study the impact of a hypothetical technology shock on the equilibrium, in particular, its effects on the average profit and the productivity threshold for entry. The analysis shows that the effects of technology shocks depend on whether the marginal cost of more productive firms is more or less sensible to the change in technology. In other words, the effect depends on the relationship between productivity and the semi-elasticity of marginal cost with respect to the technology parameter in question.

To do this, I add a generic technology parameter into the marginal cost function, i.e., $c[\phi, m(\overline{\pi}); \gamma] = \tilde{c}(\phi; \gamma)m(\overline{\pi})$, and analyze the equilibrium response to a change in this parameter.[12] For simplicity of exposition, I make an additional assumption that increasing γ raises marginal cost. The results in the opposite case can be obtained similarly.

Assumption 3. *The marginal cost function satisfies* $\frac{\partial \tilde{c}(\phi;\gamma)}{\partial \gamma} \geq 0$ *for all* ϕ *and* γ.

[11] since both ϕ^* and $\overline{\pi}$ fully characterize the equilibrium, we can use an arbitrary function $m(.)$ of average profit to represent any other average variables.

[12] I assume the functional separability as in Assumption 2. Besides, the technology parameter does not affect the function $m(.)$.

I denote the semi-elasticity as

$$e(\phi;\gamma) \equiv \frac{1}{\tilde{c}(\phi;\gamma)}\frac{\partial \tilde{c}(\phi;\gamma)}{\partial \gamma},$$

which measures how sensitive the marginal cost is to a change in the production technology. In the general case, this semi-elasticity varies with firms' productivity, which implies that heterogeneous firms have a different sensibility to a change in their technology parameter. The following result shows that the variation of this semi-elasticity across heterogeneous firms is crucial for determining the effect of a shock on the sectoral equilibrium.

Proposition 2. *Given Assumption 1, 2 and 3,*

(i) *if* $\frac{\partial e}{\partial \phi} > 0$ *(or* $\frac{\partial e}{\partial \phi} < 0$*), an increase in* γ *(with other conditions remaining the same) leads to lower (or higher)* ϕ^**,* $\overline{r}$ *and* $\overline{\pi}$*;*

(ii) *if* $\frac{\partial e}{\partial \phi} = 0$*, the effect of* γ *on* ϕ^**,* $\overline{r}$ *and* $\overline{\pi}$ *vanishes.*

Proposition 2 shows that we can determine the equilibrium effects of a change in production technology by only examining whether the marginal cost of more productive firms is more or less sensible to the change. In particular, the sign of $\frac{\partial e}{\partial \phi}$ determines the sign of $\frac{\partial \phi^*}{\partial \gamma}$, $\frac{\partial \overline{r}}{\partial \gamma}$ and $\frac{\partial \overline{\pi}}{\partial \gamma}$. The formal proofs are given in Appendix A.4. The intuition behind Proposition 2 is the following. Under the ZCP condition, given a fixed pair of σ and f, the net profit of a ϕ-firm (7) is determined by the ratio $\frac{\tilde{c}(\phi)}{\tilde{c}(\phi^*)}$ that compares its marginal costs with the one of a ϕ^*-firm. The effect of a shock on ϕ-firm's net profit depends on how responsive the marginal cost is to the change at different productivity levels. For instance, when the shock affects the marginal cost of all types of firms, in the same way, i.e., $\frac{\partial e}{\partial \phi} = 0$, then the effects are canceled out in (7). Under Assumption 2, when more productive firms are more sensitive to the shock $\frac{\partial e}{\partial \phi} > 0$, a change in the technology parameter will have a negative impact on firms' net profit. Since the technology parameter only affects the ZCP condition and does not alternate the FE condition, the changes of average profit lead to lower (or higher) ϕ^*.

5 An offshoring model

Now, I present a model where firms can relocate a part of its production to foreign countries, in order to benefit from a cheaper labor force. To model firms' offshoring activity, I follow the Groizard et al. (2014, p.226-p.227)'s approach. Considering a production that consists of a continuum of tasks in the interval $\alpha \in [0,1]$. The quantity of output produced by a $\phi-$firm is

$$q = \phi \left[\int_0^{\hat{\alpha}} q_f(\alpha)^{\frac{\rho-1}{\rho}} d\alpha + \int_{\hat{\alpha}}^1 q_d(\alpha)^{\frac{\rho-1}{\rho}} d\alpha \right]^{\frac{\rho}{\rho-1}}, \tag{10}$$

where two types of labor input are required in the production. The labor inputs are inelastically supplied from the domestic market of size L_d and the foreign market of size L_f. Firms choose an optimal level of offshoring, i.e., $\hat{\alpha}$ that the tasks in the range of $[0, \hat{\alpha}]$ are executed using the foreign labor, while the tasks in the range of $[\hat{\alpha}, 1]$ are executed using the domestic labor. For a given task α, $q_f(\alpha)$ denotes the firms' per-task requirement of foreign labor, and $q_d(\alpha)$ denotes the firms' per-task requirement of domestic labor. ρ is the elasticity of substitution between domestic labor and foreign labor inputs. I assume that one unit of foreign labor is not equivalent to one unit of domestic labor. Let $k(\alpha)$ denotes the additional cost of making foreign labor compatible with domestic production. Thus, a task α can be executed by either hiring one unit of domestic labor or $k(\alpha) > 1$ units of foreign labor. The tasks in $[0, \hat{\alpha}]$ are ordered in the way that higher indexed tasks require higher offshoring cost, which implies that $k(\alpha)$ is strictly increasing in α. Thus, the higher indexed tasks can be viewed as headquarter tasks, which are more difficult to be relocated.

Groizard et al., (2014, Lemma 1) shows that the production function can be expressed in terms of total labor demand as

$$q = \phi \left[K(\hat{\alpha})^{\frac{1}{\rho}} l_f^{\frac{\rho-1}{\rho}} + (1-\hat{\alpha})^{\frac{1}{\rho}} l_d^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{11}$$

where $K(\hat{\alpha}) = \int_0^{\hat{\alpha}} k(\alpha)^{1-\rho} d\alpha$; l_f denotes the total foreign labor demand of a ϕ-firms; l_d denotes its total domestic labor demand. This function has the CES functional form, but unlike the classic CES function its share parameters, $K(\hat{\alpha})^{\frac{1}{\rho}}$ and $(1-\hat{\alpha})^{\frac{1}{\rho}}$ depend on the optimal level of offshoring, $\hat{\alpha}$. Using the duality theorem, the marginal cost function is

$$c = \frac{1}{\phi} \left[K(\hat{\alpha}) w_f^{1-\rho} + (1-\hat{\alpha}) w_d^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{12}$$

where w_d and w_f denote the domestic and foreign wages, respectively.

Firms' cost minimizing behavior implies that cost of offshoring a task equals the cost of producing it using domestic labor, i.e., $k(\hat{\alpha}) w_f = w_d$, which determines an optimal level of $\hat{\alpha}$. The main analysis of this paper can be carried out without a specific functional form of $k(.)$. However, I assume that $k(.)$ is an exponential function of α to facilitate reading of the model. This choice also reflects that higher the indexed tasks are arduous to relocate to foreign countries, such as the managerial positions at firms' headquarter. This choice yields the following optimal offshore intensity:

$$\hat{\alpha} = \log\left(\frac{w_d}{w_f}\right). \tag{13}$$

Firms offshore only if the foreign wage is sufficiently low with respect to the domestic one. The derivation of (10), (11), (12) and (13) is reported in Appendix B.

6 Fair wage consideration

I assume that workers have a preference for fairness in the sense of Akerlof (1982). Under this fair wage consideration, workers provide an effort level above the required minimum in exchange for a fair wage, which should reflect firms' economic performance. The empirical results in Abowd et al. (1999) suggest a positive correlation between wages and firms' productivity. This may reflect the fact that heterogeneous firms face different fairness criteria, and the fair wage varies with productivity. Similar to Egger and Kreickemeier (2009), I consider a fair wage condition that has two components: one internal determinant (firms' productivity) and one external determinant (the sector-wide average wage and employment rate). The two forces are regulated by a parameter, $\theta \in [0, 1]$, which measures the tightness in the wage-productivity nexus.

$$w_d = [h(\phi)]^{\theta}[(1-U)\bar{w}_d]^{(1-\theta)}; \tag{14}$$

$$w_f = [n(\phi)]^{\theta}[(1-U)\bar{w}_f]^{(1-\theta)}, \tag{15}$$

where $h(\phi)$ and $n(\phi)$ are the wage deviation of individual firms from the sector-wide averages, $(1-U)\bar{w}_d$ and $(1-U)\bar{w}_f$. Now, I introduce a series of assumptions on the two fair wages (14) and (15).

Assumption 4.

(i) *$h(\phi)$ and $n(\phi)$ are increasing functions of ϕ, $h(\phi)/n(\phi) > 1$ is also an increasing function of ϕ;*

(ii) *$(1-U)$, the employment rate is the same in both domestic and foreign labor market;*

(iii) $\bar{w}_f = \lambda\bar{w}_d$ with $\lambda < 1$.

In a benchmark case where the market-clearing wages equal a reference wage, i.e., $w_d = w_f = w^*$, the optimal choice is to use only the domestic labor input. The model then reduces to the autarky case as in Melitz (2003). The autarky also emerges in the case where offshoring is not allowed, i.e., $\hat{\alpha} = 0$, but the fair wage condition remains, as in Egger and Kreickemeier (2009). In the former case, the autarky is due to lack of incentives, while the autarky is imposed in the latter case.

When $\theta \neq 0$, the wages vary with firms' productivity, which in turn implies that under Assumption 4 (i), the optimal offshoring level $\hat{\alpha}$ is also a function of productivity. Thus, heterogeneous firms under the fair wage consideration, face different wages and choose to offshore at different levels. This feature distinguishes the proposed model from the one in Groizard et al. (2014), where all heterogeneous firms have the same level of offshoring. Assumption 4 (i) also implies that the current model to be in line with the empirical findings (Tomiura, 2005, 2007) that more productive firms also face higher wage difference, thus prefer to offshore more to the foreign countries. From a theoretical perspective, this assumption can be justified as the following. The tasks in the production function are ordered in the way that higher indexed tasks can be viewed as headquarter tasks. Since domestic workers only handle the high indexed tasks, they are also close to the headquarter and enjoying more bargaining power in the wage setting than their foreign counterparts. To simplify exposition of the model, I impose specific functional forms on $h(.)$ and $n(.)$. For the wage of domestic workers, the fair wage condition is identical to the one in Egger and Kreickemeier (2009):

$$w_d = \phi^{\theta}[(1-U)\bar{w}_d]^{(1-\theta)}; \tag{16}$$

For the wage of foreign workers, the fair wage condition is

$$w_f = \phi^{\theta\eta}[(1-U)\bar{w}_f]^{(1-\theta)} \tag{17}$$

with $\eta < 1$. Thus, for all $\phi > 1$, these functional forms verify Assumption 4 (i).

Assumption 4 (ii) simplifies the derivation of labor market clearing condition, but it also limits this analysis to the intensive margin of labor demand–the changes in labor demand due to each firm already in the market hiring more or fewer workers. The full implications of this assumption will be discussed in Section 8. Assumption 4 (iii) implies proportionality between the domestic and foreign average wage. The parameter λ reflects a fixed "north-south" wage difference, which includes differences in living costs and skills as well as the home country's offshoring restrictions, among other things. Substituting the wages in (13) with (16) and (17), we obtain the optimal offshoring level of a given firm:

$$\hat{\alpha} = \log\left[\phi^{\theta(1-\eta)}(1/\lambda)^{1-\theta}\right], \tag{18}$$

where three parameters characterize firms' incentive to offshore their production units: the average wage difference (λ), the variable wage difference (η) and the fair wage parameter (θ). The following sections compare different equilibrium states and economic outcomes by varying these parameters. Thus, the following analysis shows how changes in the wage setting can affect firms' offshoring decision, sector-wide economic outcomes, and the labor market.

7 Comparative statics under the fair wage consideration

Under Assumption 4, equations (12) and (18) characterize a specific production model that I will examine in the rest of this paper. I start the analysis of this model by rewriting the marginal cost as

$$\begin{aligned} c &= \frac{w_d}{\phi}\left\{k[\hat{\alpha}(\phi)]^{\rho-1}K[\hat{a}(\phi)] + 1 - \hat{a}(\phi)\right\}^{\frac{1}{1-\rho}} \\ &= \frac{w_d}{\phi}[x(\phi) + 1 - \hat{a}(\phi)]^{\frac{1}{1-\rho}}, \end{aligned} \tag{19}$$

where the second equality comes from the fact that $k(.)$ is an exponential function, and $x(\phi) \equiv \frac{1-\exp[\hat{a}(\phi)]^{\rho-1}}{1-\rho} > 0$. Note that when $\theta = 0$, and the domestic wage is normalized to one, we obtain a standard marginal cost function that has been widely used in the trade literature: the marginal cost equals the inverse of productivity (Melitz, 2003). The expression (19) illustrates three channels through which productivity affects the marginal cost: a direct productivity effect, a wage effect, and an offshoring effect. Although they are not independent of each other, we can discuss them individually in order to see how productivity affects the marginal cost. First, we look that the case where firms do not have the access to the foreign labor market, i.e., $\{k[\hat{\alpha}(\phi)]^{\rho-1}K[\hat{a}(\phi)] + 1 - \hat{a}(\phi)\}^{\frac{1}{1-\rho}} = 1$. Then, the term $\frac{1}{\phi}$ captures the direct productivity effect, which means more productive firms have lower marginal cost. At the same time, the fair wage condition implies that more productive firms pay a higher wage, thus face higher marginal cost. Combining the opposite forces, we can show that overall more productive firms operate at a lower marginal cost. Now, firms are allowed to offshore, the term $\{k[\hat{\alpha}(\phi)]^{\rho-1}K[\hat{a}(\phi)] + 1 - \hat{a}(\phi)\}^{\frac{1}{1-\rho}}$ captures the offshore-induced cost reduction. Given Assumption 4, more productive firms face higher domestic wage, thus prefer to offshore more to the foreign countries where the wage is lower and less correlated with productivity ($\eta < 1$). It can be showed that $c(\phi) \in \left[\frac{w_f}{\phi}, \frac{w_d}{\phi}\right]$ for $\hat{\alpha} > 0$ and $\frac{\partial c}{\partial \hat{\alpha}} < 0$. This means that offshoring allows firms to operate at a lower marginal cost. Lemma 3 summarizes the effect of productivity on firms' optimal offshoring level and their marginal cost.

Lemma 3. *(i) More productive firms offshore more, i.e.,* $\frac{\partial \hat{\alpha}}{\partial \phi} = \frac{(1-\eta)\theta}{\phi} > 0$*; (ii) More productive firms operate at a lower marginal cost, i.e.,* $\frac{\partial c}{\partial \phi} < 0$.

In the heterogeneous firm model, a key component of the analysis is to examine how productivity is reflected in firms' production cost structure. In Melitz (2003) and many other similar studies, the marginal cost is merely inverse of productivity. Egger and Kreickemeier (2009) introduces the fair wage condition and allows wage to vary with productivity, thus opens an additional channel where productivity can influence the marginal cost. As (19) showed, the proposed model goes a step further by taking into account offshoring. One implication of this extension is that the marginal cost becomes a highly non-linear function of productivity, and improvements in productivity are non-neutral. Recalling the general results in Section 4, I now examine the equilibrium under this specific model. First, Proposition 1 and Lemma 3 (ii) indicate that the model with the specific production technology has a unique equilibrium. Then, using Proposition 2, we can examine the equilibrium impact of changes in λ, η and θ in the following sections.

7.1 Effects of increasing λ on the equilibrium

$\lambda < 1$ is the average wage difference between home and foreign countries. An increase in λ raises the average wage of foreign workers, which can result from a general increase of living standard in the foreign countries or it also can be the consequence of more restrictive offshoring rules. The current political debates in the most industrialized economies justify the study of such possibilities. For instance, President-elect Donald Trump proposed in December of 2016 to punish American firms that move their production overseas by imposing an import tariff of 35% on their sale of goods back into the United States. This will increase the cost of offshoring (or equivalently reduces the wage difference), which he believes that can keep jobs in the domestic market. The analysis of the labor effects of increasing λ is the main discussion of the next section. Here, I present the effects of increasing λ on firms' production and its implications on the product market equilibrium.

Lemma 4. *Under Assumption 4, given the production technology defined in (18) and (19), with other conditions remaining the same*

(i) *increasing* λ *leads to lower offshoring level,* $\frac{\partial \hat{a}}{\partial \lambda} = \frac{\theta-1}{\lambda} < 0$;

(ii) *increasing* λ *leads to higher marginal cost,* $\frac{\partial c}{\partial \lambda} > 0$;

(iii) $e_\lambda = \frac{1}{\tilde{c}}\frac{\partial \tilde{c}}{\partial \lambda} = -\frac{\partial \hat{a}}{\partial \lambda}\frac{x(\phi)}{x(\phi)+1-\hat{a}(\phi)} > 0$ *and* $\frac{\partial e_\lambda}{\partial \phi} > 0$

Reducing the average wage difference leads to a lower offshoring level with a constant downward slope of $\frac{\theta-1}{\lambda}$ for all firms (Lemma 4, i). Increasing λ raises the marginal production cost (Lemma 4, ii) because firms rely more on their domestic production and have to pay more to their foreign workers. Proposition 2 shows that the sector-wide effects of a particular change in the marginal cost function depend on how different firms are affected differently by this change. e_λ denotes the semi-elasticity of marginal cost with respect to λ, which is the product of two terms: $-\frac{\partial \hat{a}}{\partial \lambda} = \frac{1-\theta}{\lambda}$ and $\frac{1}{\tilde{c}}\frac{\partial \tilde{c}}{\partial \hat{a}} = \frac{x(\phi)}{x(\phi)+1-\hat{a}(\phi)}$. Although all firms response to increasing λ in the same way by reducing their offshoring level with the same slope, the reduction of offshoring level, however, will deteriorate the relative position of more productive firms with a larger increase in their marginal cost. Appendix A.6 shows that the derivative of e_λ with respect to ϕ is positive. Thus, using Proposition 2, we can obtain the following result.

Proposition 3 *Under Assumption 4, an increase of* λ *leads to lower cutoff productivity* ϕ^* *and lower average profits of active firms* $\overline{\pi}$.

7.2 Effects of increasing η on the equilibrium

$\eta < 1$ measures the correlation between a firm's productivity and the wage offered to their foreign workers. In an extreme case where $\eta = 0$, firms are insensible to the fair wage preference of their foreign workers and all firms propose the same wage. In recent years, the supply chain of multinational corporations has been the subject of public scrutiny. Under this pressure, the high profile and often more productive firms are forced to role out new management policies, such as supplier code of conduct, e.g. Apple's Supplier Responsibility Programme.[13] These policies suppose to guarantee their foreign workers a fair working condition including work safety, issuance, on the job training, among others. Adding these fringe benefits

[13]https://www.apple.com/supplier-responsibility/

to the basic wage yields a higher real wage and fairer rent sharing wage setting. Assuming that firms take their domestic working condition as a benchmark, this will lead to a catch-up of foreign workers, which can be stylized as an increasing η.

Lemma 5. *Under Assumption 4, given the production technology defined in (18) and (19), with other conditions remaining the same*

(i) *increasing η leads to lower offshoring level,* $\frac{\partial \hat{a}}{\partial \eta} = -\theta \log(\phi) < 0$;

(ii) *increasing η leads to higher marginal cost,* $\frac{\partial c}{\partial \eta} > 0$;

(iii) $e_\eta = -\frac{\partial \hat{a}}{\partial \eta} \frac{x(\phi)}{x(\phi)+1-\hat{a}(\phi)} > 0$ *and* $\frac{\partial e_\eta}{\partial \phi} > 0$

Similar to increasing λ, the rise of η leads to lower offshoring level and higher marginal cost (Lemma 5 i and ii). However, more productive firms reduce more their offshoring level in response to increasing η with the slope $-\theta \log(\phi)$. This improves the relative position of less productive firms because compared to their more productive competitor they now pay lower foreign wages, and continue to benefit from offshoring. The sector-wide effects are summarized in the following proposition. The formal proof is given in Appendix A.7.

Proposition 4. *Under Assumption 4, an increase of η leads to lower cutoff productivity ϕ^* and lower average profits of active firms $\overline{\pi}$.*

Comparing the expression of e_λ and e_η, the difference appears in how $\hat{a}$ reacts to the changes. In the case of λ, the slope is fixed for all firms, while the slope increases with productivity in the case of η. One implication is that increasing η may have more pronounced effects than increasing λ. The reason is that increasing λ or η deteriorates the relative position of more productive firms, besides, increasing η also improves the relative position of less productive firms in the sector. Consequently, increasing η will help less productive firms to survive in the market; thus the average productivity falls in this case.

7.3 Effects of increasing θ on the equilibrium

$\theta < 1$ regulates the tightness between productivity and wage for both domestic and foreign workers, and it stylizes the fairness concern. Unlike the two previous parameters, the effects of increasing θ are ambiguous. First, in an extreme case where $\theta = 0$, the wages equal the averages, thus a firm's incentive to offshore only reflects the average wage difference λ. Then, an increase in θ makes the wages gradually depart from the averages. Lemma 6 (i) states that increasing θ encourages more productive firm with $\phi > \exp[\log(1/\lambda)/(1-\eta)]$ to offshore more and reduces the offshoring level of less productive firms. Recall equation (18), we can also rewrite the expression as the sum of the logarithmic variable wage difference $\log(\phi^{1-\eta}) = \log[h(\phi)/n(\phi)]$, and the log fixed wage difference $\log(1/\lambda) = \log(\bar{w}_d/\bar{w}_f)$, weighted by θ.

$$\hat{\alpha} = \theta \log(\phi^{1-\eta}) + (1-\theta)\log(1/\lambda). \tag{20}$$

An increase in θ raises the importance of the variable wage difference in the offshoring decision, and diminish the importance of the average wage difference. For high productivity firms, this means more incentive to offshore. For low productivity firms, it is the opposite.

Lemma 6. *Under Assumption 4, given the production technology defined in (18) and (19), with other conditions remaining the same, the effects of increasing θ is ambiguous:*

(i) $\frac{\partial \hat{a}}{\partial \theta} = \log(\phi^{1-\eta}) - \log(1/\lambda)$;

(ii) $\frac{\partial c}{\partial \theta} > 0$;

(iii) $e_\theta = \log(\phi) - \frac{\partial \hat{a}}{\partial \theta}\frac{x(\phi)}{x(\phi)+1-\hat{a}(\phi)} > 0$

Lemma 6 (ii) shows that although increasing θ has an ambiguous effect on firms offshoring level; it is clear how increasing θ affects the marginal cost. In the case of high (or low) productive firms, higher θ leads to higher (lower) domestic wage, thus increases (or decreases) the marginal cost. At the same time, higher θ lead to higher (or lower) offshoring level, which in turn decreases (or increases) the marginal cost. Thus, two opposite forces influence the marginal costs. However, we can show that the wage effect dominates the offshoring effect in our case. Thus, higher θ leads to higher marginal cost.

In order to understand the expression of e_θ in Lemma 6 (iii), we can begin with a simpler version: a closed economy under the fair wage consideration (similar to Egger and Kreickemeier, 2009). In this case, the marginal cost function is $c = w_d(\phi)/\phi$ with $e_\theta = \log(\phi)$, which yields a positive derivative: $\frac{\partial e_\theta}{\partial \phi} = \frac{1}{\phi}$. In the closed economy, the marginal cost of more productive firms is always more sensible to the increase of θ. Using Proposition 2, we can find the similar result as in Egger and Kreickemeier (2009, Proposition 1) that higher θ lowers ϕ^*. However, the opening to offshoring yields an additional term in e_θ. The first term in Lemma 6 (iii), $\log(\phi)$ captures the autarky case that increasing θ raises disproportionately the domestic labor costs among heterogeneous firms with lager effects on more productive firms. At the same time, the second term $\frac{\partial \hat{a}}{\partial \theta} \frac{x(\phi)}{x(\phi)+1-\hat{a}(\phi)}$ captures that this technology shock also biases firms' factor composition in a way that more productive firms rely more on their foreign labor force, which is offered at a lower wage. This reduces the semi-elasticity of marginal cost with respect to changes in θ for high productive firms. We can then summarize the findings as the following.

Proposition 5. *In a closed economy, a higher θ leads to lower productivity cutoff ϕ^* and lower average profit $\overline{\pi}$, because this change deteriorates the relative position of more productive firms. Offshoring allows more productive firms to reduce the marginal production cost by relying more on their cheaper foreign workers. Thus the negative effect of increasing θ on ϕ^* and $\overline{\pi}$ is alleviated in the offshoring sectors.*

8 Labor market effects

In the previous section, I analyzed the equilibrium consequences of changes in key parameters. In this section, I focus on the labor market, in particular the effect of average wage difference λ on labor demand and labor composition.[14]

8.1 Labor demands at the firm-level

For a given cutoff productivity ϕ^*, the labor demands of $\phi-$firm are

$$l_d = (\sigma - 1)f \ \frac{1-\hat{\alpha}}{w_d^{\rho}} \ \phi^{\rho-1} \ \frac{c^{\rho-\sigma}}{c(\phi^*)^{1-\sigma}}; \tag{21}$$

$$l_f = (\sigma - 1)f \ \frac{K(\hat{\alpha})}{w_f^{\rho}} \ \phi^{\rho-1} \ \frac{c^{\rho-\sigma}}{c(\phi^*)^{1-\sigma}}. \tag{22}$$

The first question that can be asked here is whether more productive firms hire more (or less) domestic labor, i.e., the sign of $\frac{\partial l_d}{\partial \phi}$ in a given equilibrium. At the firm-level, keeping σ, f, and ϕ^* constant, (21) illustrates four channels that productivity influences the domestic labor demand of offshoring firms. First, there is a job reallocation effect, which is captured by $1-\hat{\alpha}$. Lemma 3 (i) shows that more productive firms offshore more, thus employ less domestic labor. Lemma 4 (i) and 5 (i) imply that a decrease in λ or η will also lead to less domestic labor via the same

[14]The same analysis can be also carried out for other parameters. I choose λ as an illustration, because the effect of λ on the equilibrium is clear cut (Proposition 3).

channel. The second and the third channel are the wage effect captured by $1/w_d^\rho$, and the pure productivity effect captured by $\phi^{\rho-1}$. The two terms show that higher domestic wage decreases the employment, and more productive firms employ more domestic labor if $\rho > 1$. The last term involving c captures a cost reduction effect. Lemma 3 (ii) indicates that c is decreasing with ϕ. Therefore, if the domestic and foreign labor are complements that $\rho - \sigma < 0$, more productive firms will employ more domestic labor via the cost reduction channel. If the domestic and foreign labor are highly substitutable that $\rho - \sigma > 0$, the opposite occurs. Finally, (21) shows that productivity affects the domestic labor demand via multiple and conflicting channels.

The question of whether more productive firms hire more (or less) domestic labor depends on the parameters and is indeterminate. Similarly, how changes in parameters affect firms' absolute number of domestic (or foreign) workers is also unclear. First, as showed in Lemma 4, 5 and 6, changes in parameter affect the level of $\hat{a}$, w_d, w_f and c, which may lead to conflicting results on labor demand. Second, any changes in parameter may lead to a new equilibrium, which affects the value of average variables $(1-U)$, $\overline{w}_d$ and $\overline{w}_f$. Thus, in the following lines, I focus on a less ambiguous variable–the relative domestic labor. Dividing domestic labor demand of a given firm by its foreign labor demand yields the labor ratio l_r, which is independent of productivity cutoff ϕ^*. Then, two clear-cut results on l_r can be obtained.

$$l_r \equiv \frac{l_d}{l_f} = \frac{1-\hat{a}}{K(\hat{a})}[\phi^{(\eta-1)\theta}\lambda^{1-\theta}]^\rho \tag{23}$$

Lemma 7. *Under Assumption 4*

(i) *More productive firms hire relative more foreign labor than domestic labor, i.e.,* $\frac{1}{l_r}\frac{\partial l_r}{\partial \phi} = -\frac{\partial \hat{a}}{\partial \phi}\left[\frac{1}{1-\hat{a}} + \frac{1}{x} + \rho\right] < 0$*;*

(ii) *Higher λ leads to relatively higher domestic labor demand, i.e.,* $\frac{1}{l_r}\frac{\partial l_r}{\partial \lambda} = -\frac{\partial \hat{a}}{\partial \lambda}\left[\frac{1}{1-\hat{a}} + \frac{1}{x} + \rho\right] > 0$

Lemma 7 (i) states that more productive firms have a lower domestic labor ratio l_r. This is because more productive firms have more incentive to offshore with $\frac{\partial \hat{a}}{\partial \phi}$ being positive (Lemma 3 i). Note that Lemma 7 (i) does not imply more productive firms hire less domestic worker in the absolute term. More productive firms rely more on their foreign workers because they are paying a higher wage to their domestic workers. Both productivity and offshoring reduce the marginal cost and allow more productive firms to charge a more competitive price. Therefore, compared to the low productive firms, it is likely that more productive firms hire more foreign as well as domestic workers in response to higher demand for their products. In the absolute term, a more productive firm may have higher domestic employment than a lower productive firm, which is often documented in the empirical studies (Bernard et al., 2003).

Lemma 7 (ii) shows that an increase in λ raises the domestic labor ratio l_r.[15] The intuition behind this result is straightforward. Higher λ means smaller wage difference between the domestic and foreign workers, which leads to lower offshoring level $\frac{\partial \hat{a}}{\partial \lambda} < 0$ (Lemma 4 i). Since l_r is independent of marginal cost c and productivity cutoff ϕ^*, an increase in λ has a direct and positive effect on l_r. This result seems to support policies that make offshoring less attractive with the objective of protecting domestic employment. However, Lemma 7 (ii) reports only the firm-level effect. In the next subsection, we shall see the sector-average effect of increasing λ, which is less obvious.

8.2 Labor demands at the sector-level

At the sector-level, a change in λ alters the equilibrium with two implications on the sector's domestic employment: the extensive and intensive margin of employment changes. The extensive margin reflects the impact of the shock on the equilibrium mass of producing firms. The intensive margin reflects the impact on the average labor demand of a firm. Here I focus on the intensive margin components of sector employment changes.

For a given productivity cutoff, the sector-average labor demands are

$$\bar{l}_j = \int_{\phi^*}^{\infty} l_j \;\; g(\phi \mid \phi \geq \phi^*) d\phi, \tag{24}$$

where the expression of l_j with $j = d, f$ are given in (21) and (22). Taking the derivative of equation above with respect to λ yields

$$\frac{\partial \bar{l}_j}{\partial \lambda} = \frac{g(\phi^*)}{1 - G(\phi^*)} \frac{\partial \phi^*}{\partial \lambda} \left[\bar{l}_j - l_j(\phi^*) \right] + \int_{\phi^*}^{\infty} \frac{\partial l_j(\phi)}{\partial \lambda} g(\phi \mid \phi \geq \phi^*) d\phi, \tag{25}$$

which shows the effect of increasing λ on the average labor demand has two components. The first term accounts for the change in employment due to the effect of λ on ϕ^*. The second term accounts for the change in employment within each firm. The question of how λ affects ϕ^* has been discussed in the previous sections. Now,

[15] Lemma 7 reports only the effects of increasing λ on labor ratio. However the similar can also be obtained for η.

in order to determine the effects of λ on $\bar{l}_j$, we need to answer addition questions, (i) are the average firm hires more (or less) j-labor than the least productive firm in the sector; (ii) how λ directly and indirectly affects the firms-level labor demand. However, both questions are unclear and depend on the interplay of various forces. Instead, in this section, I focus only on an easier target–the sector-average of labor ratio, $\overline{l_r} \equiv \frac{\overline{l_d}}{\overline{l_f}}$.

Now I introduce two alternative functions, $\tilde{l}_d$ and $\tilde{l}_f$ that satisfy:

$$\bar{l}_r = \frac{\bar{\tilde{l}}_d}{\bar{\tilde{l}}_f} = \frac{\int_{\phi^*}^{\infty} \tilde{l}_d \; g(\phi \mid \phi \geq \phi^*) d\phi}{\int_{\phi^*}^{\infty} \tilde{l}_f \; g(\phi \mid \phi \geq \phi^*) d\phi}. \tag{26}$$

Using the functional separability of the marginal cost fucntion, $\tilde{l}_d$ and $\tilde{l}_f$ are

$$\tilde{l}_d = (1-\hat{a})\phi^{\sigma(1-\theta)-1}\left[K(\hat{a})k(\hat{a})^{\rho-1} + 1 - \hat{a}\right]^{\frac{\rho-\sigma}{1-\rho}}; \tag{27}$$

$$\tilde{l}_f = K(\hat{a})\lambda^{(\theta-1)\rho}\phi^{\theta\rho(1-\eta)+\sigma(1-\theta)-1}\left[K(\hat{a})k(\hat{a})^{\rho-1} + 1 - \hat{a}\right]^{\frac{\rho-\sigma}{1-\rho}}. \tag{28}$$

Equation (27) and (28) are the domestic and foreign labor demand of a $\phi-$firm by diving the term $(\sigma-1)f \cdot c(\phi^*)^{\sigma-1}[(1-U)\overline{w}_d]^{(\theta-1)\sigma}$. The main advantage of $\tilde{l}_d$ and $\tilde{l}_f$ over the labor demands defined in (21) and (22) is that both $\tilde{l}_d$ and $\tilde{l}_f$ do not depend on any sector-averages. This reduces substantially the difficulty of this analysis.

Lemma 7 (ii) shows clearly that increasing λ raises l_r for a given firm. However, the following paragraphs will show that the effect of λ on $\bar{l}_r$ is rather ambiguous. The semi-elasticity of $\bar{l}_r$ with respects to λ is

$$\frac{1}{\bar{l}_r}\frac{\partial \bar{l}_r}{\partial \phi} = \frac{1}{\bar{\tilde{l}}_d}\frac{\partial \bar{\tilde{l}}_d}{\partial \phi} - \frac{1}{\bar{\tilde{l}}_f}\frac{\partial \bar{\tilde{l}}_f}{\partial \phi} \tag{29}$$

with

$$\frac{\partial \bar{\tilde{l}}_d}{\partial \phi} = \frac{g(\phi^*)}{1-G(\phi^*)}\frac{\partial \phi^*}{\partial \lambda}\left[\bar{\tilde{l}}_d - \tilde{l}_d(\phi^*)\right] + \int_{\phi^*}^{\infty}\frac{\partial \tilde{l}_d}{\partial \lambda} g(\phi \mid \phi \geq \phi^*) d\phi \tag{30}$$

$$\frac{\partial \bar{\tilde{l}}_f}{\partial \phi} = \frac{g(\phi^*)}{1-G(\phi^*)}\frac{\partial \phi^*}{\partial \lambda}\left[\bar{\tilde{l}}_f - \tilde{l}_f(\phi^*)\right] + \int_{\phi^*}^{\infty}\frac{\partial \tilde{l}_f}{\partial \lambda} g(\phi \mid \phi \geq \phi^*) d\phi \tag{31}$$

The effects of λ on l_r and $\bar{l}_r$ are different for two main reasons. First, increasing λ alternates the equilibrium on the product market, which affects both domestic and foreign demand negatively. Second, the labor demands differ among heterogeneous firms and vary with productivity. Thus, the effect of λ on $\bar{l}_r$ depends on the distribution of productivity as well as the variation of labor demand among firms. To illustrate the latter point, we can express the semi-elasticity $\frac{1}{\bar{l}_r}\frac{\partial \bar{l}_r}{\partial \phi}$ by ignoring the effect of λ on ϕ^*.

Assuming that $\frac{\partial \phi^*}{\partial \lambda}$ is negligible:

$$\frac{\partial \bar{\tilde{l}}_d}{\partial \phi} \simeq \int_{\phi^*}^{\infty} \frac{\partial \tilde{l}_d}{\partial \lambda} g(\phi \mid \phi \geq \phi^*) d\phi \tag{32}$$

$$\frac{\partial \bar{\tilde{l}}_f}{\partial \phi} \simeq \int_{\phi^*}^{\infty} \frac{\partial \tilde{l}_f}{\partial \lambda} g(\phi \mid \phi \geq \phi^*) d\phi \tag{33}$$

with

$$\frac{\partial \tilde{l}_d}{\partial \lambda} = -\tilde{l}_d \frac{\partial \hat{a}}{\partial \lambda} \left[(\sigma - \rho) \frac{x}{x + 1 - \hat{a}} + \frac{1}{1 - \hat{a}} \right] \tag{34}$$

$$\frac{\partial \tilde{l}_f}{\partial \lambda} = -\tilde{l}_f \frac{\partial \hat{a}}{\partial \lambda} \left[(\sigma - \rho) \frac{x}{x + 1 - \hat{a}} - \frac{1}{x} - \rho \right] \tag{35}$$

Thus, the semi-elasticity of $\bar{l}_r$ with respect to λ becomes:

$$\begin{aligned} \frac{1}{\bar{l}_r} \frac{\partial \bar{l}_r}{\partial \lambda} \quad \simeq \quad & -\frac{\partial \hat{a}}{\partial \lambda} \int_{\phi^*}^{\infty} \left[\left(\frac{1}{1 - \hat{a}} \right) \frac{\tilde{l}_d}{\bar{\tilde{l}}_d} + \left(\frac{1}{x} + \rho \right) \frac{\tilde{l}_f}{\bar{\tilde{l}}_f} \right] g(\phi \mid \phi \geq \phi^*) d\phi \\ & -\frac{\partial \hat{a}}{\partial \lambda} \int_{\phi^*}^{\infty} (\sigma - \rho) \frac{x}{x + 1 - \hat{a}} \left(\frac{\tilde{l}_d}{\bar{\tilde{l}}_d} - \frac{\tilde{l}_f}{\bar{\tilde{l}}_f} \right) g(\phi \mid \phi \geq \phi^*) d\phi \end{aligned} \tag{36}$$

Two comments can be made by comparing the firm-level and the sector-level effects of λ. First, increasing λ lower the offshoring level, which contribute positively to the relative share of domestic labor demand for both individual firms and the sector-average. Second, the terms in the integral reveals the difference between the individual and sectoral effects of λ. The first integral is the average of individual effects $\left[\frac{1}{1-\hat{a}} + \frac{1}{x} + \rho\right]$ over the sector with $\frac{\tilde{l}_d}{\bar{\tilde{l}}_d}$, $\frac{\tilde{l}_f}{\bar{\tilde{l}}_f}$ and $g(\phi \mid \phi \geq \phi^*)$ as weights. The second integral adds further individual-sector difference, which has an indeterminate sign. Note that if all firms have the same shares of domestic and foreign labor, $\frac{\tilde{l}_d}{\bar{\tilde{l}}_d} = \frac{\tilde{l}_f}{\bar{\tilde{l}}_f}$ for all ϕ, the second integral vanishes. The following proposition summarizes the main findings of Section 8.1 and 8.2.

Proposition 6. *The employment effects of increasing fixed wage difference λ on the individual firms and the sector differ. The relative share of domestic worker l_r within each firm raises as λ increases, but the effect on the sector-average of relative share $\bar{l}_r$ is ambiguous. Firm heterogeneity plays a crucial role in explaining the discrepancies between the individual and sector effects.*

8.3 Labor market equilibrium and welfare

The labor market clearing condition states that aggregate payment to workers should match the difference between aggregate revenue and profit. Given the normalization that $P = 1$ the aggregate revenue coincides with the total output. Therefore, the labor market clearing condition can be expressed as

$$Q = \left(\frac{\sigma}{\sigma - 1}\right)[\overline{w}_d(1-U)L_d + \overline{w}_f(1-U)L_f], \tag{37}$$

where L_d and L_f are the domestic and foreign labor endowment. $L = L_d + L_f$ denotes the total labor endowment. Under the assumption that the employment rate is the same in both domestic and foreign labor market, we have the following equality between the aggregate and average labor ratio:

$$\frac{L_d}{L_f} = \frac{\bar{l}_d}{\bar{l}_f} = \bar{l}_r \tag{38}$$

The additional assumption that the average domestic and foreign wages are proportional, $\overline{w}_f = \lambda\overline{w}_d$, allows us to write the per capita output as:

$$\frac{Q}{L} = \frac{\sigma}{\sigma - 1}\overline{w}_d(1-U)\left(\frac{\bar{l}_r + \lambda}{\bar{l}_r + 1}\right) \tag{39}$$

The mass of producing firms in any period can then be determined as:

$$M = \frac{\overline{w}_d(1-U)L}{(\sigma - 1)(\overline{\pi} + f)}\left(\frac{\bar{l}_r + \lambda}{\bar{l}_r + 1}\right) \tag{40}$$

Given that L is exogenously fixed, we only need to express $\overline{w}_d(1-U)$ to complete the characterization of the unique stationary equilibrium. The intersection of pricing rule (4) and the fair wage condition (14) with $h(.)$ being a linear and $\phi = \overline{\phi}$ yields

$$\overline{w}_d(1-U) = \left(\frac{\sigma - 1}{\sigma}\right)^{\frac{1}{1-\theta}} \overline{\phi}[x(\overline{\phi}) + 1 - \hat{a}(\overline{\phi})]^{\frac{-1}{(1-\rho)(1-\theta)}}. \tag{41}$$

Following Egger and Kreickemeier (2009); Egger et al. (2015), we can use the per capita output as the general measure of utilitarian welfare. Equation (39) shows that this welfare measure depends on the elasticity of substitution between the varieties, σ; the domestic labor market condition $\overline{w}_d(1-U)$, the average relative share of domestic labor, $\bar{l}_r$ and the fixed wage difference λ. It is to note that, since the foreign workers have a lower average wage ($\lambda < 1$), a higher $\bar{l}_r$ contributes positively the general welfare. Now, I separate the general welfare for the domestic and foreign workers. I define the domestic income as $Q_d = \left(\frac{\sigma}{\sigma-1}\right) \overline{w}_d(1-U)L_d$ and the foreign income as $Q_f = \left(\frac{\sigma}{\sigma-1}\right) \overline{w}_f(1-U)L_f$. Thus, the domestic welfare is proportional to the foreign welfare, and can be expressed as

$$\frac{Q_d}{L_d} = \lambda \frac{Q_f}{L_f} = \left(\frac{\sigma-1}{\sigma}\right)^{\frac{\theta}{1-\theta}} \overline{\phi}[x(\overline{\phi}) + 1 - \hat{a}(\overline{\phi})]^{\frac{-1}{(1-\rho)(1-\theta)}}, \tag{42}$$

which shows how a increasing λ affects the welfare measures.

Proposition 7. *A reduction of fixed wage difference between domestic and foreign workers, (i) equalizes the domestic and foreign welfare, and (ii) causes a decline of domestic utilitarian welfare.*

One may argue that the utilitarian welfare measure only reflects the efficiency of the economy, and may not be featured as the top priority of the trade unions in the collective bargaining. the unions may be more concerned about the actual transfer of revenue to the domestic workers, i.e., $\overline{w}_d\bar{l}_d$. Using the accounting identity $(1-U)L = M(\bar{l}_d + \bar{l}_f)$, the employment rate is given as

$$(1-U) = \frac{1}{r(\overline{\phi})} \left(\frac{\sigma-1}{\sigma}\right)^{\frac{\theta}{1-\theta}} \overline{\phi}[x(\overline{\phi}) + 1 - \hat{a}(\overline{\phi})]^{\frac{-1}{(1-\rho)(1-\theta)}} (\bar{l}_d + \lambda \bar{l}_f) \tag{43}$$

Dividing (41) by (43) yields the average domestic wage $\overline{w}_d$. Then, the average payment to the domestic workers can be obtianed by multiplying $\bar{l}_d$:

$$\overline{w}_d\bar{l}_d = \frac{\sigma-1}{\sigma} \frac{r(\overline{\phi})}{1+\lambda/\bar{l}_r}. \tag{44}$$

This equation shows that, on the one hand, an increasing λ can lower the average productivity and revenue, which contributes negatively to $\overline{w}_d\bar{l}_d$. On the other hand, if increasing λ can raise the average share of domestic labor, it can also contribute positivity to $\overline{w}_d\bar{l}_d$. Thus, the overall effect of increasing λ on the average domestic payment results from a combination of opposite reactions.

9 Final remarks

Following the empirical evidence, this paper has outlined a new framework for analyzing firms' offshoring behavior and the corresponding labor market implications under the fair wage concern. The proposed model incorporates three critical elements of an open economy: heterogeneous firms, labor market frictions, and offshoring. In this paper, I first derived a series of results based on an unspecified function to illustrate the equilibrium implications of production technology explicitly. I found that how a change in technology affects the economy depends on whether the marginal cost of more productive firms is more or less sensible to the technology shock in question. Then, I apply the general results to the model of interest with specific production technology.

I consider an offshoring model to characterize an economy with heterogeneous firms, which have access to the international labor markets and may choose to offshore certain activities in order to minimize their domestic labor costs. I then introduce a fair wage condition to the offshoring model that both domestic and foreign wages vary with firms' productivity. In this wage setting mechanism, a fairness parameter regulates the general tightness in the wage-productivity nexus, and two parameters explain the domestic and foreign wage differences. This model extends the classic models of the heterogeneous firm in the sense that productivity is not merely a Hicks-neutral technical term, but it can also affect many aspects of the production organization. Using this model, I examine the effects of changes in the wage setting on the individual firm behavior as well as sectoral equilibrium. The finding shows that a change in the wage setting affects heterogeneous firms' offshoring incentive, which can lead to a shift of sectoral equilibrium. The crucial variable that regulates the effects is the semi-elasticity of marginal production cost with respect to the shock in question. The analysis of labor market illustrate the differences between individual and sector effects: a shock that can help to increase the domestic employment of some firms may not be able to generate the domestic employment gain at the sector-level.

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A Proofs of lemmas and propositions

A.1 Proof of Lemma 1

Using the implicit function theorem, we have

$$\frac{\partial \overline{\phi}}{\partial \phi^*} = -\frac{\partial F / \partial \phi^*}{\partial F / \partial \overline{\phi}} \tag{A.1}$$

with

$$F(\overline{\phi}, \phi^*) = \int_{\phi^*}^{\infty} \left[\frac{c(\phi)}{c(\overline{\phi})}\right]^{1-\sigma} g(\phi \mid \phi \geq \phi^*) d\phi - 1 \tag{A.2}$$

The derivatives of F are

$$\frac{\partial F}{\partial \phi^*} = \frac{g(\phi^*)}{1 - G(\phi^*)} \left\{1 - \left[\frac{c(\phi^*)}{c(\overline{\phi})}^{1-\sigma}\right]\right\}; \tag{A.3}$$

$$\frac{\partial F}{\partial \overline{\phi}} = \frac{\sigma - 1}{c(\phi^*)} \frac{\partial c(\overline{\phi})}{\partial \overline{\phi}}. \tag{A.4}$$

Given Assumption 1 that $\frac{\partial c(\overline{\phi})}{\partial \overline{\phi}} < 0$, $\frac{\partial F}{\partial \phi^*} > 0$ and $\frac{\partial F}{\partial \overline{\phi}} < 0$, which implies Lemma1.

A.2 Proof of Lemma 2

By definition, we have

$$\overline{r} = \int_{\phi^*}^{\infty} r(\phi) g(\phi \mid \phi \geq \phi^*) d\phi \tag{A.5}$$

and we know that

$$r(\phi) = \left[\frac{c(\phi)}{c(\overline{\phi})}\right]^{(1-\sigma)} r(\overline{\phi}). \tag{A.6}$$

Thus,

$$\overline{r} = r(\overline{\phi}) \int_{\phi^*}^{\infty} \left[\frac{c(\phi)}{c(\overline{\phi})}\right]^{(1-\sigma)} g(\phi \mid \phi \geq \phi^*) d\phi = r(\overline{\phi}) \tag{A.7}$$

which proves Lemma 2 (i). Since $\pi(\phi) = r(\phi)/\sigma$, Lemma 2 (ii) can be proved in a similar way.

A.3 Proof of Proposition 1

We define two functions of ϕ^*:

$$\hat{\pi}(\phi^*) = \left[\frac{c(\overline{\phi})}{c(\phi^*)}\right]^{(1-\sigma)} - 1; \tag{A.8}$$

$$j(\phi^*) = [1 - G(\phi^*)]\hat{\pi}(\phi^*). \tag{A.9}$$

Under Assumption 1, $\hat{\pi}(\phi^*) > 0$, $j(\phi^*) > 0$ and $\lim_{\phi^* \to \infty} j(\phi^*) = 0$. First, I determine the derivative of $\hat{\pi}(\phi^*)$ w.r.t ϕ^*. Given the definition of $\overline{\phi}$, $\hat{\pi}(\phi^*)$ can be rewritten as

$$\hat{\pi}(\phi^*) = \frac{1}{1 - G(\phi^*)} \frac{1}{c(\phi^*)^{(1-\sigma)}} \int_{\phi^*}^{\infty} c(\phi)^{(1-\sigma)} g(\phi) d\phi - 1. \tag{A.10}$$

Thus,

$$\frac{\partial \hat{\pi}(\phi^*)}{\partial \phi^*} = \frac{g(\phi^*)}{1 - G(\phi^*)} \hat{\pi}(\phi^*) - (1-\sigma)[\hat{\pi}(\phi^*) + 1] \frac{1}{c(\phi^*)} \frac{\partial c(\phi^*)}{\partial \phi^*}. \tag{A.11}$$

Then,

$$\begin{aligned} \frac{\partial j(\phi^*)}{\partial \phi^*} &= -g(\phi^*)\hat{\pi}(\phi^*) + [1 - G(\phi^*)] \frac{\partial \hat{\pi}(\phi^*)}{\partial \phi^*} \\ &= (\sigma - 1)[1 - G(\phi^*)][\hat{\pi}(\phi^*) + 1] \frac{1}{c(\phi^*)} \frac{\partial c(\phi^*)}{\partial \phi^*}. \end{aligned} \tag{A.12}$$

Given Assumption 1, we have $\frac{\partial j(\phi^*)}{\partial \phi^*} < 0$, which proves this proposition. Under Assumption 2, the result in Proposition 1 remains, which can be proved simplify by replacing $c(.)$ with $\tilde{c}(.)$.

A.4 Proof of Proposition 2

Part I: Combining ZCP and FE conditions, we have the following implicit function:

$$F(\phi^*, \gamma) = j(\phi^*; \gamma) - \frac{\delta f_e}{f} = 0 \tag{A.13}$$

Using the implicit fucntion theorem, we can obtain

$$\frac{\partial \phi^*}{\partial \gamma} = -\frac{\frac{\partial F}{\partial \gamma}}{\frac{\partial F}{\partial \phi^*}} = -\frac{[1 - G(\phi^*)]}{f} \frac{\frac{\partial \overline{\pi}}{\partial \gamma}}{\frac{\partial j}{\partial \phi^*}} \tag{A.14}$$

The proof in Proposition 1 shows that $\frac{\partial j(\phi^*)}{\partial \phi^*} < 0$, thus $\frac{\partial \phi^*}{\partial \gamma}$ and $\frac{\partial \overline{\pi}}{\partial \gamma}$ have the same sign.

Part II: Under the functional separability of the marginal cost fucntion (Assumption 2), the definition of average productivity also implies:

$$\int_{\phi^*}^{\infty} \left[\frac{\tilde{c}(\phi)}{\tilde{c}(\overline{\phi})} \right]^{1-\sigma} g(\phi \mid \phi \geq \phi^*) d\phi = 1. \tag{A.15}$$

The derivative of average profit w.r.t the technology parameter is

$$\begin{aligned} \frac{\partial \overline{\pi}}{\partial \gamma} = &\ (\sigma - 1) f \left[\frac{\tilde{c}(\overline{\phi}; \gamma)}{\tilde{c}(\phi^*; \gamma)} \right]^{(1-\sigma)} \\ &\left[\frac{1}{\tilde{c}(\phi^*; \gamma)} \frac{\partial \tilde{c}(\phi^*; \gamma)}{\partial \gamma} - \frac{1}{\tilde{c}(\overline{\phi}; \gamma)^{(1-\sigma)}} \int_{\phi^*}^{\infty} \tilde{c}(\phi; \gamma)^{-\sigma} \frac{\partial \tilde{c}(\phi; \gamma)}{\partial \gamma} g(\phi \mid \phi \geq \phi^*) d\phi \right] \end{aligned} \tag{A.16}$$

Focusing on the case where $\frac{\partial \tilde{c}(\phi;\gamma)}{\partial \gamma} > 0$, which implies $e(\phi;\gamma) > 0$.

First, in the case of $\frac{\partial e(\phi;\gamma)}{\partial \phi} > 0$, and given Assumption 1 and 2, both $\tilde{c}(\phi,;\gamma)^{1-\sigma}$ and $e(\phi;\gamma)$ are increasing with ϕ. Therefore the covariance between the two variable is positive. Now, we can simplify the notation using $\mathrm{E}_{|\phi \geq \phi^*}[f(\phi)] \equiv \int_{\phi^*}^{\infty} f(\phi) g(\phi \mid \phi \geq \phi^*) d\phi$. Using the property of conditional covariance, we have:

$$\mathrm{E}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma} e(\phi;\gamma)] = \mathrm{E}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma}] \cdot \mathrm{E}_{|\phi\geq\phi^*}[e(\phi;\gamma)] + \mathrm{Cov}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma} e(\phi;\gamma)] \tag{A.17}$$

Thus,

$$\mathrm{E}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma} e(\phi;\gamma)] > \mathrm{E}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma}] \cdot \mathrm{E}_{|\phi\geq\phi^*}[e(\phi;\gamma)] \tag{A.18}$$

Then, multiplying both sides of the inequality above by a positive value $\frac{1}{\mathrm{E}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma}]}$ yields

$$\frac{\mathrm{E}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma} e(\phi;\gamma)]}{\mathrm{E}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma}]} > \mathrm{E}_{|\phi\geq\phi^*}[e(\phi;\gamma)] \tag{A.19}$$

which can be rewritten as

$$\frac{1}{\tilde{c}(\overline{\phi};\gamma)^{(1-\sigma)}} \int_{\phi^*}^{\infty} \tilde{c}(\phi;\gamma)^{-\sigma} \frac{\partial \tilde{c}(\phi;\gamma)}{\partial \gamma} g(\phi \mid \phi \geq \phi^*) d\phi > \int_{\phi^*}^{\infty} \frac{1}{\tilde{c}(\phi;\gamma)} \frac{\partial c(\phi;\gamma)}{\partial \gamma} g(\phi \mid \phi \geq \phi^*) d\phi \tag{A.20}$$

Using the monotonicity of the conditional expected value, we know that

$$\int_{\phi^*}^{\infty} \frac{1}{\tilde{c}(\phi;\gamma)} \frac{\partial \tilde{c}(\phi;\gamma)}{\partial \gamma} g(\phi \mid \phi \geq \phi^*) d\phi > \int_{\phi^*}^{\infty} \frac{1}{\tilde{c}(\phi^*;\gamma)} \frac{\partial \tilde{c}(\phi^*;\gamma)}{\partial \gamma} g(\phi \mid \phi \geq \phi^*) d\phi \tag{A.21}$$

Finally, we have

$$\frac{1}{\tilde{c}(\overline{\phi};\gamma)^{(1-\sigma)}} \int_{\phi^*}^{\infty} \tilde{c}(\phi;\gamma)^{-\sigma} \frac{\partial \tilde{c}(\phi;\gamma)}{\partial \gamma} g(\phi \mid \phi \geq \phi^*) d\phi > \frac{1}{\tilde{c}(\phi^*;\gamma)} \frac{\partial \tilde{c}(\phi^*;\gamma)}{\partial \gamma} \tag{A.22}$$

which implies $\frac{\partial \overline{\pi}}{\partial \gamma} < 0$ and $\frac{\partial \phi^*}{\partial \gamma} < 0$.

Second, in the case of $\frac{\partial e(\phi;\gamma)}{\partial \phi} < 0$, the proof is similar. Now, we have

$$\mathrm{E}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma} e(\phi;\gamma)] < \mathrm{E}_{|\phi\geq\phi^*}[\tilde{c}(\phi;\gamma)^{1-\sigma}] \cdot \mathrm{E}_{|\phi\geq\phi^*}[e(\phi;\gamma)] \tag{A.23}$$

Then,

$$\begin{aligned} &\frac{1}{\tilde{c}(\overline{\phi};\gamma)^{(1-\sigma)}} \int_{\phi^*}^{\infty} \tilde{c}(\phi;\gamma)^{-\sigma} \frac{\partial \tilde{c}(\phi;\gamma)}{\partial \gamma} g(\phi \mid \phi \geq \phi^*) d\phi \\ &\quad < \int_{\phi^*}^{\infty} \frac{1}{\tilde{c}(\phi;\gamma)} \frac{\partial \tilde{c}(\phi;\gamma)}{\partial \gamma} g(\phi \mid \phi \geq \phi^*) d\phi \\ &\quad < \frac{1}{\tilde{c}(\phi^*;\gamma)} \frac{\partial \tilde{c}(\phi^*;\gamma)}{\partial \gamma} \end{aligned} \tag{A.24}$$

which implies $\frac{\partial \overline{\pi}}{\partial \gamma} > 0$ and $\frac{\partial \phi^*}{\partial \gamma} > 0$. Finally, given Lemma 2 that $\overline{\pi} = \frac{\overline{r}}{\sigma} - f$, thus we can apply the similar results for revenues.

A.5 Proof of Lemma 3

(i) The derivative of $\hat{\alpha}$ with respect to ϕ is

$$\frac{\partial \hat{\alpha}}{\partial \phi} = \frac{\theta(1-\eta)}{\phi} > 0.$$

(ii) Let H denotes the term $\left[k(\hat{\alpha})^{\rho-1}K(\hat{\alpha}) + (1-\hat{\alpha})\right]^{\frac{1}{1-\rho}}$, the derivation of c with respects to ϕ:

$$\frac{\partial c}{\partial \phi} = \frac{\partial w_d/\phi}{\partial \phi} H + \frac{w_d}{\phi}\frac{\partial H}{\partial \phi}$$

First, given that $\theta < 1$, we have

$$\frac{\partial w_d/\phi}{\partial \phi} = (\theta-1)\phi^{\theta-2}[(1-U)\overline{w}_d]^{1-\theta} < 0$$

Then, given that $k(.)$ is an increasing function, we can derive

$$\frac{\partial H}{\partial \phi} = -\left[k(\hat{\alpha})^{\rho-1}K(\hat{\alpha}) + (1-\hat{\alpha})\right]^{\frac{\rho}{1-\rho}} k(\hat{\alpha})^{\rho-2}K(\hat{\alpha})\frac{\partial k(\hat{\alpha})}{\partial \hat{\alpha}}\frac{\partial \hat{\alpha}}{\partial \phi} < 0$$

Note that Lemma 3(ii) is proved without imposing a specific functional forms on $k(.)$.

A.6 Proof of Lemma 4 and Corollary 1

(i) The derivative of $\hat{\alpha}$ with respect to λ is

$$\frac{\partial \hat{\alpha}}{\partial \lambda} = \frac{\theta - 1}{\lambda} < 0.$$

(ii) Now, I use a fact that $k(.)$ is an exponential function. Then c can be rewritten as:

$$c = \frac{w_d}{\phi}[x(\phi) + 1 - \hat{\alpha}(\phi)]^{\frac{1}{1-\rho}} \tag{A.25}$$

where $x(\phi) = \frac{1-\exp[\hat{\alpha}(\phi)]^{\rho-1}}{1-\rho} > 0$. Then, the derivation of c with respects to λ:

$$\begin{aligned}\frac{\partial c}{\partial \lambda} &= \frac{w_d}{\phi}\frac{1}{1-\rho}[x(\phi)+1-\hat{\alpha}(\phi)]^{\frac{\rho}{1-\rho}}\left(\frac{\partial x}{\partial \lambda} - \frac{\partial \hat{a}}{\partial \lambda}\right) \\ &= -\frac{w_d}{\phi}[x(\phi)+1-\hat{\alpha}(\phi)]^{\frac{\rho}{1-\rho}}x(\phi)\frac{\partial \hat{a}}{\partial \lambda} > 0\end{aligned}$$

which proves Lemma 4 (ii).

(iii) Under the functional separability $c = [(1-U)\overline{w}_d]^{1-\theta}\tilde{c}$, where $\tilde{c} = \phi^{\theta-1}[x(\phi) + 1 - \hat{\alpha}(\phi)]^{\frac{1}{1-\rho}}$. The derivation of $\tilde{c}$ with respects to λ:

$$\frac{\partial \tilde{c}}{\partial \lambda} = -\phi^{\theta-1}[x(\phi) + 1 - \hat{\alpha}(\phi)]^{\frac{\rho}{1-\rho}} x(\phi) \frac{\partial \hat{a}}{\partial \lambda} > 0 \tag{A.26}$$

Defining $e_\lambda = \frac{1}{\tilde{c}} \frac{\partial \tilde{c}}{\partial \lambda}$, the derivative of e_λ w.r.t ϕ is

$$\frac{\partial e_\lambda}{\partial \phi} = \frac{1-\theta}{\lambda} \frac{\left\{exp[\hat{a}(\phi)]^{\rho-1}[1-\hat{a}(\phi)] + x(\phi)\right\}}{[x(\phi) + 1 - \hat{\alpha}(\phi)]^2} \frac{\partial \hat{a}}{\partial \phi} > 0 \tag{A.27}$$

A.7 Proof of Lemma 5 and Corollary 2

(i) The derivative of $\hat{\alpha}$ with respect to η is

$$\frac{\partial \hat{\alpha}}{\partial \eta} = -\theta \log(\phi) < 0.$$

(ii) The derivation of c with respects to η:

$$\frac{\partial c}{\partial \eta} \quad = \quad -\frac{w_d}{\phi}[x(\phi) + 1 - \hat{\alpha}(\phi)]^{\frac{\rho}{1-\rho}} x(\phi) \frac{\partial \hat{a}}{\partial \eta} > 0$$

(iii) Defining $e_\eta = \frac{1}{\tilde{c}} \frac{\partial \tilde{c}}{\partial \eta}$, the derivative of e_η w.r.t ϕ is

$$\frac{\partial e_\eta}{\partial \phi} = \frac{\theta x(\phi)}{\phi[x(\phi) + 1 - \hat{a}(\phi)]} + \theta \log(\phi) \frac{\left\{\exp[\hat{a}(\phi)]^{\rho-1}[1-\hat{a}(\phi)] + x(\phi)\right\}}{[x(\phi) + 1 - \hat{\alpha}(\phi)]^2} \frac{\partial \hat{a}}{\partial \phi} > 0$$

B Production technology of an offshoring firm

B.1 The production function

The final good production consists of two intermediate productions: the foreign production with tasks ranged between 0 and $\hat{\alpha}$, and the domestic production with tasks ranged between $\hat{\alpha}$ and 1. The domestic production is characterized by a linear technology: $q_d(\alpha) = l_d^\alpha(\alpha)$, where $l_d^\alpha(\alpha)$ is the per-task labor requirement of any tasks $\alpha \in [\hat{\alpha}, 1]$. This implies that two distinctive tasks require the same quantities of labor:

$$l_d^\alpha(\alpha) = l_d^\alpha(\alpha') \text{ for } \alpha, \alpha' \in [\hat{\alpha}, 1]. \tag{B.1}$$

Equation (B.1) characterizes the optimal allocation of domestic labor across different tasks that labor has the same productivity across tasks and same wage across tasks.

Unlike the domestic production, there is an extra cost of making foreign labor compatible with the task α. Thus, the foreign production is characterized by $q_f(\alpha) = \frac{l_f^\alpha(\alpha)}{k(\alpha)}$, where $l_f^\alpha(\alpha)$ is the per-task foreign labor requirement of any task $\alpha \in [0, \hat{\alpha}]$. The optimal allocation condition, $\frac{\partial q}{\partial q_f} = k(\alpha)$, implies:

$$k(\alpha) = \phi \left[\int_0^{\hat{\alpha}} q_f(\alpha)^{\frac{\rho-1}{\rho}} d\alpha + \int_{\hat{\alpha}}^1 q_d(\alpha)^{\frac{\rho-1}{\rho}} d\alpha \right]^{\frac{\rho}{\rho-1}-1} q_f(\alpha)^{-\frac{1}{\rho}}$$

By comparing two different tasks, we obtain

$$\frac{q_f(\alpha)}{q_f(\alpha')} = \left[\frac{k(\alpha')}{k(\alpha)} \right]^{\rho}$$

Replace $q_f(\alpha)$ and $q_f(\alpha')$ in the previous equation by $l_f^\alpha(\alpha)$ and $l_f^\alpha(\alpha')$, which yields the foreign labor requirement of two distinctive tasks, $\alpha, \alpha' \in [0, \hat{\alpha}]$:

$$\frac{l_f^\alpha(\alpha)}{l_f^\alpha(\alpha')} = \left[\frac{k(\alpha)}{k(\alpha')} \right]^{1-\rho} \quad \text{for } \alpha, \alpha' \in [0, \hat{\alpha}]. \tag{B.2}$$

Using (B.1), we can express the total demand for domestic labor, l_d as

$$l_d = \int_{\hat{\alpha}}^1 l_d^\alpha(\alpha) d\alpha = \int_{\hat{\alpha}}^1 l_d^\alpha(\alpha') d\alpha = l_d^\alpha(\alpha')(1-\hat{\alpha}),$$

which implies that

$$l_d^\alpha(\alpha) = \frac{l_d}{1-\hat{\alpha}} \qquad \forall \alpha \in [\hat{\alpha}, 1]. \tag{B.3}$$

Using (B.2) with $\alpha' = \hat{\alpha}$, we can write the total demand for foreign labor l_f as

$$l_f = \int_0^{\hat{\alpha}} l_f^\alpha(\alpha) d\alpha = \int_0^{\hat{\alpha}} \left[\frac{k(\alpha)}{k(\hat{\alpha})} \right]^{1-\rho} l_f^\alpha(\hat{\alpha}) d\alpha = \frac{\int_0^{\hat{\alpha}} k(\alpha)^{(1-\rho)} d\alpha}{k(\hat{\alpha})^{1-\rho}} l_f^\alpha(\hat{\alpha})$$

which implies that

$$l_f^\alpha(\alpha) = \left[\frac{k(\alpha)}{k(\hat{\alpha})} \right]^{1-\rho} l_f^\alpha(\hat{\alpha}) = \frac{k(\alpha)^{1-\rho} l_f}{\int_0^{\hat{\alpha}} k(\alpha)^{(1-\rho)} d\alpha} \tag{B.4}$$

Substituting $q_f(\alpha)$ and $q_d(\alpha)$ in the production function by l_d and l_f yields the production function of offshoring firms:

$$\begin{aligned} q &= \phi \left\{ \int_0^{\hat{\alpha}} \left[\frac{l_f^\alpha(\alpha)}{k(\alpha)} \right]^{\frac{\rho-1}{\rho}} d\alpha + \int_{\hat{\alpha}}^1 l_d^\alpha(\alpha)^{\frac{\rho-1}{\rho}} d\alpha \right\}^{\frac{\rho}{\rho-1}} \\ &= \phi \left\{ {}^{\frac{1-\rho}{\rho}} \left[\int_0^{\hat{\alpha}} k(\alpha)^{1-\rho} d\alpha \right]^{\frac{1}{\rho}} l_f^{\frac{\rho-1}{\rho}} + (1-\hat{\alpha})^{\frac{1}{\rho}} l_d^{\frac{\rho-1}{\rho}} \right\}^{\frac{\rho}{\rho-1}} \end{aligned}$$

B.2 The marginal cost function and the optimal offshoring level

For a given level of productivity, firms minimize the total cost of production:

$$\min_{l_f(\phi),l_d(\phi)} w_f l_f(\phi) + w_d l_d(\phi)$$

subject to the production function. The first order conditions of the minimization problem are

$$w_f = \mu\phi \left\{ K(\hat{\alpha})^{\frac{1}{\rho}} l_f(\phi)^{\frac{\rho-1}{\rho}} + (1-\hat{\alpha})^{\frac{1}{\rho}} l_d^{\frac{\rho-1}{\rho}} \right\} K(\hat{\alpha})^{\frac{1}{\rho}} l_f(\phi)^{\frac{-1}{\rho}};$$

$$w_d = \mu\phi \left\{ K(\hat{\alpha})^{\frac{1}{\rho}} l_f(\phi)^{\frac{\rho-1}{\rho}} + (1-\hat{\alpha})^{\frac{1}{\rho}} l_d^{\frac{\rho-1}{\rho}} \right\}^{\frac{1}{\rho-1}} (1-\hat{\alpha})^{\frac{1}{\rho}} l_d(\phi)^{\frac{-1}{\rho}},$$

where μ is the Lagrange multiplier. Combining the two first order conditions yields

$$\frac{w_f}{w_d} = \frac{K(\hat{\alpha})^{\frac{1}{\rho}} l_f(\phi)^{\frac{-1}{\rho}}}{(1-\hat{\alpha})^{\frac{1}{\rho}} l_d(\phi)^{\frac{-1}{\rho}}}$$

This in turn gives the input share equation:

$$\frac{l_f(\phi)}{l_d(\phi)} = \frac{K(\hat{\alpha})}{1-\hat{\alpha}} \left(\frac{w_d}{w_f}\right)^{\rho} \tag{B.5}$$

We can also use (B.5) to obtain the marginal cost function. Replacing the term $l_f(\phi)$ in the production function (10) by $\frac{K(\hat{\alpha})}{1-\hat{\alpha}} \left(\frac{w_d}{w_f}\right)^{\rho} l_d(\phi)$ yields

$$\begin{aligned} q(\phi) &= \phi \left\{ K(\hat{\alpha})(1-\hat{\alpha})^{\frac{1-\rho}{\rho}} \left[\frac{w_d}{w_f}\right]^{\rho-1} l_d(\phi)^{\frac{\rho-1}{\rho}} + (1-\hat{\alpha})^{\frac{1}{\rho}} l_d(\phi)^{\frac{\rho-1}{\rho}} \right\}^{\frac{\rho}{\rho-1}} \\ &= \phi \left[K(\hat{\alpha}) w_f^{1-\rho} + (1-\hat{\alpha}) w_d^{1-\rho} \right]^{\frac{\rho}{\rho-1}} (1-\hat{\alpha})^{-1} w_d^{\rho} l_d(\phi) \end{aligned}$$

which gives

$$l_d(\phi) = \frac{(1-\hat{\alpha}) w_d^{-\rho}}{\left[K(\hat{\alpha}) w_f^{1-\rho} + (1-\hat{\alpha}) w_d^{1-\rho} \right]^{\frac{\rho}{\rho-1}}} \frac{q(\phi)}{\phi} \tag{B.6}$$

Similarly

$$l_f(\phi) = \frac{K(\hat{\alpha}) w_f^{-\rho}}{\left[K(\hat{\alpha}) w_f^{1-\rho} + (1-\hat{\alpha}) w_d^{1-\rho} \right]^{\frac{\rho}{\rho-1}}} \frac{q(\phi)}{\phi} \tag{B.7}$$

This allows us to write the total cost function of offshoring firms:

$$C = \left[K(\hat{\alpha}) w_f^{1-\rho} + (1-\hat{\alpha}) w_d^{1-\rho} \right]^{\frac{1}{1-\rho}} \frac{q(\phi)}{\phi} \tag{B.8}$$

Then, we determine the optimal choice of $\hat{\alpha}$ by minimizing the marginal cost. The first order condition, $\frac{\partial C}{\partial \hat{\alpha}} = 0$, yields

$$\frac{\partial K(\hat{\alpha})}{\partial \hat{\alpha}} w_f^{1-\rho} - w_d^{1-\rho} = 0,$$

which defines the optimal level of offshoring (13). Then, substituting $w_f^{1-\rho}$ in the total cost function by $\frac{w_d^{1-\rho}}{k(\hat{\alpha})^{1-\rho}}$, and dividing $q(\phi)$ yields the marginal cost fucntion:

$$c(\phi) = \frac{w_d}{\phi} \left[k(\hat{\alpha})^{\rho-1} K(\hat{\alpha}) + (1-\hat{\alpha}) \right]^{\frac{1}{1-\rho}}. \tag{B.9}$$

Finally, using (3), (4) and the ZCP condition, we can rewrite the domestic labor demand of offshoring firms.

B.3 Equilibrium labor demands at the firm-level

Combing the Marshallian demand (3) and the pricing rule (4) yields:

$$q(\phi) = \frac{\sigma - 1}{\sigma} c(\omega)^{-1} r(\phi). \tag{B.10}$$

Using the ZCP condition, the revenue can be expressed as:

$$r(\phi) = \left[\frac{c(\phi)}{c(\phi^*)} \right]^{(1-\sigma)} \sigma f. \tag{B.11}$$

Finally, substituting (B.10) and (B.11) into (B.6) and (B.7) yields the domestic and foreign labor demands of a ϕ-firm in Section 8.1.