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The Ramsey Cooperative and Non-Cooperative Unconventional Monetary Policy

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Abstract

I study the Ramsey problem for three unconventional monetary policies in a two-country model. An equity injection into financial intermediaries is the most efficient policy. Due to precautionary effects of future risk, a central bank should exit from these policies in accordance with but slower than the speed of deleveraging in the financial sector. The optimal policy is changed considerably if cross-country policy cooperation is not imposed. In this case, the unconventional interventions tend to be too strong in one country but too weak in the other. The cooperation gain is a function of policy cost. At last, I evaluate several simple rules and find that the rule responding to gaps in asset prices mimics the optimal policy very well.

JEL: E44, E58, F41, F42, C63.

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The Ramsey Cooperative and Non-Cooperative Unconventional Monetary Policy

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I study the Ramsey problem for three unconventional monetary policies in a two-country model. An equity injection into financial intermediaries is the most efficient policy. Due to precautionary effects of future risk, a central bank should exit from these policies in accordance with but slower than the speed of deleveraging in the financial sector. The optimal policy is changed considerably if cross-country policy cooperation is not imposed. In this case, the unconventional interventions tend to be too strong in one country but too weak in the other. The cooperation gain is a function of policy cost. At last, I evaluate several simple rules and find that the rule responding to gaps in asset prices mimics the optimal policy very well.

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I. Introduction

The recent financial crisis involved a significant disruption to financial intermediation. Such a disruption can propagate internationally via integrated financial markets. To stabilize the financial system, fiscal and monetary authorities in major economies acted jointly and introduced new policy tools. These policies included a provision of liquidity at a large scale, and resulted in a large balance sheet of central banks worth 20 percent to 30 percent of GDP, exposing central banks to private sector risks. Hence, they break away from traditional policy tools and are commonly known as unconventional monetary policies (UMPs)[1]. In the last ten years, different sets of UMPs have been used at different timings. For example, the Federal Reserve mainly used an expanded discount window in the early stage of the crisis. After the Lehmann failure, the Fed started its asset purchase programs (quantitative easing, or QE) and injected equities into the financial system. The Fed started to taper the QE at the end of 2013, and ceased it in October 2014, after which the Federal

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[1]These policies may not be strictly monetary policy. I use the terms unconventional policy and unconventional monetary policy interchangeably.

Reserve kept the size of its balance sheet constant by buying just enough to replace maturing securities. The European Central Bank initialized its asset purchase programs at a relatively small scale slightly later than the Federal Reserve. The ECB formally introduced QE in 2015 and further increased the amount purchased in the late 2016. The ECB also used equity injections during this period.

Although much work has been devoted to evaluating the effectiveness of the UMP (for example, Negro et al., 2017; Quint and Rabanal, 2017), the normative analysis is still missing, namely, what is the optimal way to conduct the unconventional policy especially when the foreign country may take a free-ride of domestic policy. The normative analysis does not only provide a natural benchmark against which we can evaluate certain policy implementation but also sheds some light on critical policy decisions, such as the selection of unconventional tools, the need for cross-country cooperation, and when to begin and end interventions. Since the UMPs may not be permanent, the exit from the UMPs is particularly interesting after a decade of the crisis because central banks start to discuss shrinking their large balance sheet. For instance, in a blog article, Bernanke (2017) argues from a policy communications perspective that the shrinkage should be done in a passive and predictable way.

In this paper, I study the Ramsey problem for unconventional policy and aim to address three questions. First, which one of three unconventional policies, namely public asset purchases, discount window lending, and equity injections into banks, is most effective. More generally, what are the factors that affect policy effectiveness. Second, how should the optimal policy response to a foreign or domestic shock that may trigger a global financial crisis. Then, how should a central bank exits from the policy. Third, what is the difference between the cooperatives policy coordinated across countries and the uncooperative policy conducted strategically by independent central banks. To this aim, I consider financial frictions developed by Gertler and Kiyotaki (2010) in a two-country model. The model is designed to capture key features of the recent crisis. By 2007, it was normal for a bank to have both commercial bank business and investment bank business. The banks with a high leverage are vulnerable to shocks having negative impacts on the value of their assets and to financial shocks that tighten their balance sheet. Given such shocks, the potential for insolvency makes it difficult for banks to raise short-term credit, which creates credit crunches and lead to a collapse in asset prices and investment. With financial market integration, the credit crunches immediately propagate to the foreign financial sector. In the model, the insolvency is represented by an occasionally binding incentive constraint that is derived from an agency problem between the banks and their depositors. The constraint is slack in normal times but binds endogenously in a financial crisis. The occasionally binding constraint (OBC) setting may capture data better by introducing asymmetry (Swarbrick, Holden and Levine, 2017) and by eliminating the financial accelerator mechanism during normal times (Del Negro, Hasegawa and Schorfheide, 2016). In this paper, the OBC setting also means that the unconventional policy may be conducted only occasionally when the constraint binds. More importantly, the risk of the constraint being binding in the future can have important implications for optimal policy.

My main findings are as follows. First, unconventional policy effectively crowds out private funds in the financial sector. The crowding out effect is mitigated if the policy relaxes the financial constraint that banks face. The most efficient policy is the one with the smallest crowding out effect, i.e., equity injections. Second, domestic and foreign policy respond asymmetrically to shocks. The degree of asymmetry depends on the nature of the shock, the cost of interventions, and the bank's portfolio. After a relatively strong response in the same period when the shock hits (i.e., in a non-prudential manner), the central bank exits from their interventions in accordance with the financial sector deleveraging, the speed of which depends again on the crowding out effect. Overall, due to precautionary effects of the OBC setting, the exit is slow and lasts even after the economy has escaped from the financial constraints. Third, the difference between cooperative and noncooperative policy is a function of the intervention cost. The cooperation gain is zero if the intervention cost is small. Increasing the intervention cost beyond a certain point lets the cooperation gain jump to a positive number. In the noncooperative equilibrium, the interventions are to be too strong in one country but too weak in the other.

As is well known, the Ramsey solution is silent to implementation. I proceed to compare several simple feedback rules. I find that a rule that responds to gaps in asset prices can approximate the optimal results very well. However, this rule requires knowledge of the asset prices that would be realized in a frictionless world. Among other rules that respond to observable variables, the autoregressive rule targeting at the expected credit spread gives the best results.

This paper is most closely related to Dedola, Karadi and Lombardo (2013), which focuses on the international dimension of unconventional policy. They study simple rules for public asset purchases in an economy where the incentive constraints are always binding. In their context, welfare gain of cooperation exists even the unconventional policy is costless, which is in sharp contrast to my results. My discussion about the exit from unconventional intervention is linked to Foerster (2015) who also suggests slowly unwinding the central bank's balance sheet. However, Foerster (2015) only examines simple rules for public asset purchases in a closed economy. I show that a policy with smaller crowding out effects can exit relatively faster. He and Krishnamurthy (2013) compare multiple policies: borrowing subsidies, equity injections, and public asset purchases. They also find equity injections lead to the fastest recovery. Their comparison is based on particular policy schemes that correspond to actual policies employed during the financial crisis. Gieck (2014) compares how liquidity injections and asset swaps affect the foreign exchange rate. Other relevant papers include Ellison and Tischbirek (2014) and Sheedy (2016), which study simple rules for both the conventional and unconventional policy in a closed economy. They show that the conventional and unconventional policy must be coordinated appropriately to stabilize output and inflation. Quint and Rabanal (2017) study optimal rules for public asset purchases conditional on a estimated Taylor rule and consider the effectiveness of unconventional policy in normal times. This paper is also related to the literature of occasionally binding financial constraints, for which a short review can be found in Swarbrick, Holden and Levine (2017).

The rest of the paper is organized as follows. The next section presents a two-country model with occasionally binding financial constraints. After describing my numerical method in section III, sections IV and V report the main results for cooperative and noncooperative policy respectively. I evaluate the performance of simple rules in section VI. The last section concludes the paper.

II. The Model

The model mostly follows Dedola, Karadi and Lombardo (2013), which extends the two-country real business cycle model (Backus, Kehoe and Kydland, 1992) by including a Gertler and Karadi (2011) style financial friction. The world economy consists of two countries, Home and Foreign, that are symmetric before being hit by a shock. In each country, domestic labour and capital are used to produce homogeneous goods, which can be used for consumption and capital production. To finance their capital, goods producers borrow from banks. Banks receive deposits from households in both countries and lend to goods producers in both countries. The assumption of deposit market integration may appear ad-hoc. However, relaxing this assumption only changes results mildly up to a misallocation of household savings. On the other hand, the security market integration is a key channel through for shock propagation. Overall, I follow Dedola, Karadi and Lombardo (2013) and focus on the case of fully integrated financial markets. I use the term "non-financial sector" to refer to households and producers of goods and capital, and the term "financial sector" to refer to banks. The problem facing each agent in the Home economy is described in this section. Foreign variables are denoted with "*". Lower case letters denote individual variables and real prices while upper case letters denote aggregate variables and nominal prices.

A. *Households*

There is a unit-continuum of infinitely lived households. Households consume homogeneous goods, supply labour, and save. The menu of assets available to households includes a deposit in domestic banks, $d_{h,t}$, a deposit in foreign banks, $d_{f,t}$, and a domestic government bond, b_t. Without loss of generality, only domestic citizens can hold their own government's bonds. All these assets are risk-free one-period bonds denominated in terms of the issuing country's goods and paying gross real return, r_t or, r_t^*. Households also hold shares in non-financial firms.

Each household consists of workers and bankers who pool consumption risk perfectly. Workers provide labour to goods producers and bring wages to the household. Bankers manage a bank and transfer the profits to the household when they exit from the business. It is convenient to assume that households do not save in their own banks. The complete consumption insurance allows me to write down the problem facing the consolidated household. The representative household chooses consumption, c_t, labour supply, l_t, and end-of-period wealth, consisting of domestic bonds, $d_{h,t}+b_t$ and foreign bonds, $d_{f,t}$ to maximize the expected discounted life-time utility, taking the wage rate and the interest rate as given:

$$E_t \sum_{j=0}^{\infty} \beta_{t,t+j} \left[\frac{(c_{t+j} - hc_{t+j-1})^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{1+\varphi}}{1+\varphi} \right],$$

where $h > 0$ is the habit parameter, $\sigma > 0$ is the coefficient of relative risk aversion, $\varphi > 0$ is the inverse of the (Frisch) elasticity of labour supply, $\chi \geq 0$ is the relative utility weight of labor, and $\beta_{t,t+j}$ is the subjective discount factor from period t to $t+j$. To induce stationarity of the model with incomplete financial markets, the discount factor is assumed to depend on aggregate consumption relative to aggregate income: $\beta_{t,t+1} = \bar{\beta} + \psi_\beta \log\left(\frac{C_t}{Y_t}\right)$. Denoting Π_t as the net profit distribution that the household earns from its ownership of banks and non-financial firms, w_t as the real wage rate, $\tau_{w,t}$ as a tax rate on wages, T_t as a lump-sum tax, the household faces the budget constraint

$$c_t + d_{h,t} + b_t + d_{f,t} = (1 - \tau_{w,t})\, w_t l_t + \Pi_t + (d_{h,t-1} + b_{t-1})\, r_{t-1} + d_{f,t-1} r_{t-1}^* - T_t.$$

Let $\Xi_{t,t+1} \equiv \beta_{t,t+1} \frac{(c_{t+1} - hc_t)^{-\sigma} - \beta_{t+1,t+2} h (c_{t+2} - hc_{t+1})^{-\sigma}}{(c_t - hc_{t-1})^{-\sigma} - \beta_{t,t+1} h (c_{t+1} - hc_t)^{-\sigma}}$ be the stochastic discount factor, The first-order conditions for the household problem are fairly standard:

$$w_t (1 - \tau_{w,t}) = \frac{\chi l_t^{\varphi}}{(c_t - hc_{t-1})^{-\sigma} - \beta_{t,t+1} h (c_{t+1} - hc_t)^{-\sigma}},$$

$$E_t \left[\Xi_{t,t+1} r_t\right] = 1, \tag{1}$$

$$E_t \left[\Xi_{t,t+1} r_t^*\right] = 1. \tag{2}$$

Equations 1 and 2 implies that the risk free rate is equalized across countries thanks to deposit market integration.

B. *Non-financial firms*

There are two types of non-financial firms: capital producers and goods producers.

Goods producers. — Good producers hire workers and purchase capital from capital producers to produce final goods that are homogeneous across countries. They operate in markets for goods, capital, and labour that are perfectly competitive. The production technology is a standard Cobb-Douglas function $y_t = A_t (\xi_t k_{t-1})^{\alpha} l_t^{1-\alpha}$ where α is capital share, A_t is total factor productivity, and k_t is the capital stock at the end of period t. Using δ to denote the depreciation rate, and ξ_t to govern the quality of capital, a goods producer acquires additional capital $i_t = k_t - (1-\delta)\,\xi_t k_{t-1}$ at a given price q_t. To finance its physical investment, the goods producer borrows from banks by issuing securities

$$q^s_{t,t}\left(s_t - s_{t-1}\right) = i_t q_t, \tag{3}$$

where s_t denotes the number of securities issued at the end of period t and $q^s_{T,t}$ is the period T price of security issued at period t. Each security is a state-contingent claim to the future return from one unit of investment: z_{t+1}, $(1-\delta)\,\xi_{t+1}z_{t+2}$, $(1-\delta)^2\,\xi_{t+1}\xi_{t+2}z_{t+3}$, ... with z_t denoting the gross profit per unit of capital.

The problem faced by the representative goods producer is

$$\begin{gathered}\max_{\{l_{t+j},k_{t+j},s_{t+j}\}_{j=0}^{\infty}} E_t \textstyle\sum_{j=0}^{\infty} \Xi_{t,t+j} \\ \times\left[(1-\tau_{y,t})\,y_{t+j} - w_{t+j}l_{t+j} - i_{t+j}q_{t+j} + q^s_{t+j,t+j}\left(s_{t+j}-s_{t+j-1}\right) - z_{t+j}s_{t+j-1}\right]\end{gathered},$$

subject to equation 3, the production function, and the capital accumulation equation. $\tau_{y,t}$ is a sales tax. Let the multipliers associated with equation 3 be λ^{nf}_t, the first-order conditions for the goods producer's problem are then

$$w_t = (1-\alpha)\,\frac{y_t\left(1-\tau_{y,t}\right)}{l_t},$$

$$q_t\left(1+\lambda^{nf}_t\right) = E_t\Xi_{t,t+1}\left[\frac{\partial y_{t+1}}{\partial k_t} + (1-\delta)\,\xi_{t+1}q_{t+1}\left(1+\lambda^{nf}_{t+1}\right)\right], \tag{4}$$

$$q^s_t\left(1+\lambda^{nf}_t\right) = E_t\Xi_{t,t+1}\left[z_{t+1} + q^s_{t+1}\left(1+\lambda^{nf}_{t+1}\right)\right]. \tag{5}$$

It is important to assume that investment is fully financed by securities, i.e., $\lambda^{nf}_t \neq 0$. Otherwise, firms can effectively borrow directly from households by paying a negative dividend, which make the banking sector trivial. Using equations 4 and 5, it is easy to show that the time T price of security issued at time t is $q^s_{T,t} = q_T\,(1-\delta)^T \prod_{j=1}^{T}\xi_{t+j}$, and $s_t = k_t$, given $s_0 = k_0$. Hence, I can write the return of holding security issued in period t from t to $t+1$ as

$$r_{k,t+1} = \frac{z_{t+1} + (1-\delta)\,\xi_{t+1}q_{t+1}}{q_t},$$

where the gross profit per unit of capital is obtained using the zero profit condition

$$z_t = \frac{y_t - w_t l_t}{k_{t-1}} = \alpha\frac{y_t\left(1-\tau_{y,t}\right)}{k_{t-1}}.$$

Capital producers. — Given the demand for new capital i_t and the market price q_t, capital producers maximize their expected profit discounted by the household's stochastic discount factor

$$\max_{\{i_{t+j}\}_{j=0}^{\infty}} E_t \sum_{j=0}^{\infty} \Xi_{t,t+j} \left[q_{t+j} i_{t+j} - f\left(k_{t+j-1}, i_{t+j}\right)\right],$$

subject to the cost function

$$f\left(\cdot\right) = i_t + \frac{\eta}{2}\left(\frac{i_t}{\delta k_{t-1}} - 1\right)^2 \delta k_{t-1}.$$

The first-order condition for the production decision pins down the market price of new capital

$$q_t = 1 + \eta\left(\frac{i_t}{\delta k_{t-1}} - 1\right).$$

C. Banks

Banks receive deposits amounting to $d_{h,t}$ and $d^*_{h,t}$ from domestic and foreign depositors, respectively, and purchase $s_{h,t}$ and $s_{f,t}$ units of securities from domestic and foreign goods producers. The lending channel from banks to goods producers is frictionless. Banks efficiently evaluate their assets and have a perfect enforcement on the obligations of issuers. If banks have a difficulty in raising deposits, they have the option of borrowing $d_{g,t}$ from the central bank's discount window at the interest rate $r_{g,t}$, or they may receive equity injections from their government. By holding long-term risky securities funded by short-term risk free deposits, banks in this model act as investment banks as well as commercial banks, which is a stylized fact of the recent financial crisis. For the same reason, the literature often refers to such banks as financial intermediaries.

The balance sheet of a representative bank is given by

$$\omega_t \equiv q_t s_{h,t} + q^*_t s_{f,t} = d_{h,t} + d^*_{h,t} + d_{g,t} + e_{h,t} + e_{g,t}, \tag{6}$$

where ω_t denotes the total assets of the bank, and $e_{h,t}$ and $e_{g,t}$ are equities held by households and the government respectively. The profit of banking, referred to as net worth n_t, is given by

$$e_{h,t} + e_{g,t} = n_t \equiv q_{t-1} s_{h,t-1} r_{k,t} + q^*_{t-1} s_{f,t-1} r^*_{k,t} - d_{h,t-1} r_{t-1} - d^*_{h,t-1} r^*_{t-1} - d_{g,t-1} r_{g,t-1}. \tag{7}$$

I refer to deposits, household equities as private funds, refer to discount window lending and government equities as public funds. I define bank leverage as

$$\phi_t = \frac{\omega_t - d_{g,t} - e_{g,t}}{e_t}.$$

Following Gertler and Karadi (2011), I assume that the bank shuts down with a probability $r_{n,t}$ at the end of each period and will distribute its net worth evenly to all the equities. Then, the banker becomes a worker. In the meantime, a similar number of workers from the same household randomly become new bankers. New bankers receive "start-up" funds from their household at a proportion ϖ of the total assets owned by the representative incumbent. The probability of a shutdown has two roles. First, an infinitely lived bank will sooner or later accumulate enough net worth to finance its investment without borrowing from households. In this case, the financial constraint that I detail shortly plays no role. Second, the probability enters the bank's stochastic discount factor, which ensures that the bank is always "less patient" than households so that funds always flow from households to banks. The notation of $r_{n,t}$ follows the suggestion of Swarbrick, Holden and Levine (2017) that the shutting down probability can be interpreted as an exogenous dividend rate[2]. Further more, I allow the dividend rate being affected by policy, as described in subsection II.D.

The bank choose $s_{h,t}$, $s_{f,t}$, $d_{h,t}$, $d^*_{h,t}$, and $d_{g,t}$, given prices and rates of returns, to maximize the expected present value of net worth paid upon closure

$$\begin{aligned} V_t(n_t) &= \max E_t \sum_{j=0}^{\infty} r_{n,t+j,t+j}\left(1 - r_{n,t,t+j-1}\right)\Xi_{t,t+j+1}\left(n_{t+1+j}\right) \\ &= \max E_t \Xi_{t,t+1}\left[r_{n,t,t} n_{t+1} + \left(1 - r_{n,t,t}\right)V_{t+1}\left(n_{t+1}\right)\right] \\ &= \nu_{n,t} n_t, \end{aligned} \tag{8}$$

where the third equality follows a conjecture that the value function is linear in net worth, $(1 - r_{n,i,j})$ is the probability that the bank operates until the end of period j conditional on the bank operating at the beginning of period i. The bank's ability to raise deposits is restricted up to an incentive constraint (or financial constraint)

$$OBC_t \equiv \nu_{n,t}\left(n_t - e_{g,t}\right) - \left[\theta_t\left(\omega_t - \theta_g d_{g,t}\right) - \nu_{n,t} e_{g,t}\right] \geq 0 \tag{9}$$

where OBC_t stands for an occasionally binding constraint and measures the distance of this constraint from binding, $\theta_t \in [0,1]$ and $\theta_g \in [0,1]$ are (time-varying) parameters. The intuition behind this constraint is the following. Banks are able to declare bankruptcy and exit. In this case, the banker diverts to his or her family a proportion θ_t of the divertable assets, $\omega_t - \theta_g d_{g,t}$. However, the diverted assets

[2]Swarbrick, Holden and Levine (2017) find that, when the financial constraint does not bind in the steady state, the model of Gertler and Karadi (2011) should be modified to allow endogenous dividend payments in order to match data. I find this is not crucial in a two-country model.

must firstly be used to repay government equities in full. The creditors can reclaim only the un-diverted funds. Therefore, creditors are willing to lend to a bank only if the bank has no incentive to default, i.e., the value of the private equities is larger than the value of default. The fact that $\theta_g d_{g,t}$ is un-divertable and government equities are fully secured suggests that the central bank has superior power to enforce repayment.

For convenience, the decision on $s_{h,t}$ and $s_{f,t}$ can be written in terms of the total assets ω_t and the portfolio $\alpha_{p,t} = \frac{q_t^* s_{f,t}}{\omega_t}$. Denoting the multiplier associated with inequality 9 by $\lambda_t \geq 0$, the necessary conditions of the maximization include the slackness condition of inequality 9, and the first-order conditions with respect to the total assets, the portfolio, and the borrowing from the government

$$\begin{array}{rcl} E_t \Xi_{t,t+1} \left(r_{n,t,t} + \left(1 - r_{n,t,t}\right) \nu_{n,t+1} \right) \left(r_{k,t+1} - r_t \right) & \equiv & \nu_{\omega,t} \\ & = & \frac{\lambda_t}{1+\lambda_t} \theta_t \end{array}, \tag{10}$$

$$\begin{array}{rcl} E_t \Xi_{t,t+1} \left(r_{n,t,t} + \left(1 - r_{n,t,t}\right) \nu_{n,t+1} \right) \left(r_{k,t+1} - r^*_{k,t+1} \right) & \equiv & \nu_{\alpha_p,t} \\ & = & 0 \end{array}, \tag{11}$$

$$\begin{array}{rcl} E_t \Xi_{t,t+1} \left(r_{n,t,t} + \left(1 - r_{n,t,t}\right) \nu_{n,t+1} \right) \left(r_{g,t} - r_t \right) & \equiv & \nu_{d_g,t} \\ & = & \frac{\lambda_t}{1+\lambda_t} \theta_t \theta_g \end{array}, \tag{12}$$

where in equation 10 I use the fact that $\nu_{\alpha_p,t} = 0$ for all t thanks to market integration. Given $\nu_{n,t+1} \geq 1$, the extra term $\left(r_{n,t,t} + \left(1 - r_{n,t,t}\right) \nu_{n,t+1} \right)$ multiplying the stochastic discount factor suggests that banks are generally less patient than households. The unknown time-varying coefficient in the value function can be solved using first-order conditions and the incentive constraint:

$$\nu_{n,t} = \nu_t \left(\frac{\nu_{\omega,t}}{\theta_t - \nu_{\omega,t}} + 1 \right), \tag{13}$$

where $\nu_t \equiv E_t \Xi_{t,t+1} \left(r_{n,t,t} + \left(1 - r_{n,t,t}\right) \nu_{n,t+1} \right) r_t$ is defined similarly as $\nu_{\omega,t}$, $\nu_{\alpha_p,t}$, and $\nu_{d_g,t}$. Since $\nu_{n,t}$ is independent to all the control variables of the bank, I verify the early conjecture of the linear value function.

The properties of the bank's problem have been well discussed in the literature, see Gertler and Kiyotaki (2010); Gertler and Karadi (2011); Gertler, Kiyotaki and Queralto (2012); Dedola, Karadi and Lombardo (2013). Here I only highlight some key results related to the occasionally binding constraints. To begin, use equations 6, 7, and 8 to write the value function as

$$\nu_{n,t} n_t = \nu_{\omega,t} \omega_t - \nu_{\alpha_p,t} \omega_t \alpha_{p,t} - \nu_{d_g,t} d_{g,t} + \nu_t n_t.$$

Then, $\nu_{n,t}$, $\nu_{\omega,t}$, $\nu_{\alpha_p,t}$, $\nu_{d_g,t}$, ν_t can be conveniently interpreted as the expected marginal value of net worth, total assets, portfolio, borrowing from the government, and deposits, respectively. If the financial constraint is not binding, then $\lambda_t = 0$ and the first-order conditions imply that having one extra unit of ω_t by borrowing from households or the government does not raise the bank's value. In addition, equation 13 becomes $\nu_{n,t} = \nu_t \approx 1$, meaning that net worth and deposits are valued equally at the margin. If the financial constraint binds, $\lambda_t > 0$ implies $\nu_{\omega,t} > 0$, $\nu_{d_g,t} > 0$, and $\nu_{n,t} > \nu_t$. Securities, borrowing from the government, and equities are more valuable than deposits because they also help relax the financial constraint. In addition, $\nu_{\omega,t} > 0$ indicates a credit spread between returns on securities and returns on deposits. Defining $spread_t \equiv E_t\left(r_{k,t+1} - r_t\right)$, the spread is a convenient measure of financial market effectiveness. However, unless the model is solved with certainty equivalence, the spread is positive even when the financial constraint does not bind.

D. *Government and unconventional policies*

Following the standard approach in the public finance literature, the specific agency that implements the unconventional policies is abstracted from the model. A consolidated government budget is given by

$$G_t + \Gamma_{i,t} + r_t B_{t-1} + AP_{i,t} = T_t + \tau_{w,t} w_t L_t + \tau_{y,t} Y_t + B_t + \mathcal{L}_{i,t},$$

where G_t and T_t are lump-sum government spending and tax, respectively, B_t are government bonds, $\Gamma_{i,t}$, $AP_{i,t}$, and $\mathcal{L}_{i,t}$ are resource costs, aggregate spending, and the gross profit of unconventional policy. One of three unconventional policies is available to the government, namely, public asset purchases, lending to banks, and equity injections, indexed by $i = 1, 2, 3$. The aggregate spending is therefore respectively the total asset purchased, the total discount window lending, and the value of government equities. All policies can be financed by government bonds (or reserves, the liabilities of the central bank), a lump-sum tax, a labor income tax, or a sales tax. As argued in Del Negro and Sims (2015), in order to avoid central bank insolvency, it would be appropriate for a central bank conducting unconventional policies to receive fiscal backing from the fiscal authority. I now proceed to describe each policy.

Using public asset purchases, the government lends directly to domestic goods producers[3]. Hence the government acts like a financial intermediary but faces no constraint in addition to its budget. The banks only need to fulfill the goods producers' remaining demand for funds . However, I assume that the government does not purchase foreign securities for political reasons or due to a very high cost of evaluation and monitoring[4]. If the government lends to banks via the discount window,

[3]Quint and Rabanal (2017) discuss how the model can be modified such that the asset purchases are applied to long-term government bonds

[4]There is a natural upper bound that the government can not buy more assets than those available on

replacing one unit deposit by one unit government lending relaxes the financial constraint, thanks to the government's superior power of enforcement. Consequently, the banks can expand their investment to the extent allowed by the relaxed financial constraint. Government equities stabilize the financial sector in a similar way. The constraint-relaxing effect of one unit equity, held either by households or the government, is multiplied by the marginal value of net worth $\nu_{n,t}$, which is high in a financial crisis. While private equities can only be accumulated slowly, the stock of government equities is freely adjustable. Clearly, these three policies may work against each other. For example, banks that receive equity injections would wish to expand their asset holding. However, they may not able to do so if the government also conduct a large scale asset purchase program. In this paper, I consider one policy at a time.

Before proceeding, I briefly compare the policies discussed above to those in the literature. I model public asset purchases and lending to banks following Gertler and Karadi (2011), but I model equity injections differently. Gertler and Karadi (2011) assume that a unit of government equity has the same payout stream as a unit of security. The government is willing to pay a higher price than the prevailing market price of securities. It is also assumed that government equities are non-divertable. Due to these assumptions, equity injections is effectively public asset purchases with a lump-sum transfer to banks, and hence they have very similar effects on the economy. By contrast, I assume that government equities are identical to private equities in nature, which makes this policy similar to its counterpart in He and Krishnamurthy (2013). Negro et al. (2017) consider two types of assets. The private assets are illiquid and can be sold up to a certain fraction of holding in each period. The government bonds and money, on the other hand, are liquid and not subject to this constraint. Therefore, the unconventional policy in their paper is to sell liquid assets and buy illiquid assets, roughly in line with the evolution of the asset side of the Federal Reserve balance sheet during the crisis. The illiquid assets is similar to securities in this paper. Thus, the asset swap policy in Negro et al. (2017) can be seen as a mix of discount window lending and public asset purchases in this paper.

So far, using unconventional policy is costless to the economy. Any level of interventions between just offsetting the financial friction and fully crowding out private funds are equally optimal. In the literature, the cost is often abstracted from the analysis (Negro et al., 2017; Quint and Rabanal, 2017). In my analyses of optimal policy, policy trade-offs have important implications. Following Gertler and Karadi (2011); Dedola, Karadi and Lombardo (2013); Foerster (2015) the government must pay a reduced form resource costs on its holding of securities, equities, and its landing to banks:

$$\Gamma_{i,t} = \tau A P_{i,t}^2. \tag{14}$$

the market. We are unlikely to hit this bound with reasonable calibration.

Such costs represents inefficient public activism in private financial markets or the costs of strengthened financial surveillance[5]. To facilitate comparison across policies, I assume the same τ for each policy. However, we should keep in mind that the intervention costs are arguably smaller for high-grade instruments like commercial papers, agency debt and mortgage backed securities (Gertler and Kiyotaki, 2010). Further more, the policy may suffer from a moral hazard such that the bank receiving bailout tends to pay higher dividends or to default. The moral hazard is captured by the equations below:

$$\ln r_{n,t} = \ln \bar{r_n} + \ln \left(AP_{i,t}+1\right) \psi_{r_n}, \tag{15}$$

$$\ln \theta_t = \ln \tilde{\theta}_t + \ln \left(AP_{i,t}+1\right) \psi_{\theta}, \tag{16}$$

where $\tilde{\lambda}_t$ is an exogenous stochastic process, and $\bar{r_n}$, ψ_{r_n}, and ψ_θ are positive parameters.

In appendix B, I consider the policy cost as a distorting effect of the tax that is necessary to financial at least a proportion of the policy spending. I conclude that the distortionary tax is too expensive to finance the unconventional policy. A comprehensive investigation of how the unconventional policy is financed and the associated cost requires another paper and at least some empirical evidence.

E. *Aggregation and the market clearing conditions*

The law of motion for the aggregate equities hold by households is given by

$$\begin{aligned} E_t &= \theta_{t,t} \frac{E_{t-1}}{E_{t-1}+E_{g,t-1}} \left[q_{t-1} S_{h,t-1} r_{k,t} + q^*_{t-1} S_{f,t-1} r^*_{k,t} - D_{h,t-1} r_{t-1} - D^*_{h,t-1} r^*_{t-1} - D_{g,t-1} r_{g,t-1}\right. \\ &+ \varpi \omega_{t-1}, \end{aligned}$$

where the last term is the start-up funds received by new banks. Finally, the model is closed by market clearing conditions on the goods and security markets

$$Y_t + Y^*_t = C_t + C^*_t + G_t + G^*_t + \Gamma_t + \Gamma^*_t + f\left(K_{t-1}, I_t\right) + f\left(K^*_{t-1}, I^*_t\right),$$

$$q_t S_t = q_t \left(S_{h,t} + S^*_{h,t}\right) + AP_{1,t},$$
$$q^*_t S^*_t = q^*_t \left(S_{f,t} + S^*_{f,t}\right) + AP^*_{1,t}.$$

[5]Dedola, Karadi and Lombardo (2013) add also a linear term to the cost but find only the coefficient on the quadratic term playing an important role. In my context, the linear term implies a positive marginal cost regardless the level of intervention so the Ramsey policy is nonzero even when the financial constraints do not bind. So I choose to make the cost pure quadratic.

III. The numerical method

A. Simulation method

To deal with the occasionally binding constraints (OBCs). — Stochastic models with OBCs are typically simulated using global methods. However, the model I describe above and the corresponding model to solve the Ramsey policy contain too many state variables to be simulated even by methods that are explicitly designed to deal with large state space, such as that of Maliar and Maliar (2015). Guerrieri and Iacoviello (2015) provide a fast algorithm based on piecewise linearization, which gives, certainty equivalent results, however. I employ the approach lately proposed by Holden (2016*b*,*a*). This approach supports second order approximation to evaluate welfare and captures the risk of constraint binding in the future. Dynare-OBC[6] created by the same author is a toolkit to implement this approach, which roughly consists of the following steps. First, the model is Taylor approximated up to a chosen order around the deterministic steady state. All inequalities are ignored during the approximation, but enter the approximated model. Then, the approximated model with OBCs can be solved under perfect foresight using Holden (2016*b*)'s algorithm. We can simulate a stochastic version of the model using the idea of extended path (EP) algorithm of (Fair and Taylor, 1983). For model that is linear apart from the OBCs (due to first order approximation), the simulation is certainty equivalent. For model that is non-linear apart from the OBCs (due to higher order approximation), the simulation captures the risk stemming from non-OBC nonlinearity so the slopes of variables' responses change at the bound. To further capture the risk of hitting the bound in the future, Holden (2016*a*) applied modified version of the stochastic extended path (SEP) algorithm of (Adjemian and Juillard, 2013). To form expectations, the SEP algorithm involves integrating the model over certain number of periods of future uncertainty. I integrate over 50 periods and find no considerable change if integrating over longer periods. I refer to the solutions based on EP and SEP algorithm as EP alike and SEP alike solutions respectively. And I will compare these two solutions to show the precautionary motives to avoid the bound.

To deal with the indeterminate portfolio. — An issue related to the perturbation based method is indeterminacy of the equilibrium portfolio $\alpha_{p,t}$. According to Devereux and Sutherland (2011), a second (third, fourth, ...) order approximation of the model is generally enough to pin down up to zero (first, second, ...) order term(s) of the portfolio, while the terms of the portfolio up to first (second, third, ...) order are relevant for the second (third, fourth, ...) order approximated model. The zero-order term is the deterministic steady state. Devereux and Sutherland (2011) propose a general solution as follows. Conjecturing $\alpha_{p,t}$ as a (N-1)th order

[6]DOI: http://dx.doi.org/10.5281/zenodo.50132.

polynomial of state variables in the model, we can use this conjecture to replace equation 11. Then we can simulate the model with Nth order approximation and search parameters in the conjecture such that the (N+1)th order approximation of the risk-sharing condition is satisfied.

As in Dedola, Karadi and Lombardo (2013), I only solve the zero order portfolio. This is sufficient when I focus on dynamics and calculate a first-order approximation to the model. In the evaluation of welfare, the first order terms of the portfolio are neglected as it is very demanding to compute portfolio dynamics in a model with OBCs.

B. Parameterization

TABLE 1—PARAMETERIZATION

Description	Parameter	Value
Steady-state discount factor	β	0.99
Elasticity of discount factor	ψ_β	-0.001
Habit	h	0.815
Risk aversion	σ	1.5
Weight on disutility of labour	χ	3.4
Inverse elasticity of labour supply	φ	0.276
Capital share	α	0.33
Inverse elasticity of investment to the capital price	η	1.728
Depreciation rate	δ	0.025
Steady-state survival probability of banks	$1-\bar{r}_n$	0.972
Transfer rate from households to new banks	ϖ	0.0045
Steady-state fraction of divertable assets	$\bar{\theta}$	0.2457
Fraction of un-divertable discount window borrowing	θ_g	1
Reduced form policy costs	τ	0.0001
Persistence of financial shock	ρ_θ	0.8
Standard deviation of financial shock	σ_θ	0.1
Persistence of capital quality shock	ρ_ξ	0.66
Standard deviation of capital quality shock	σ_ξ	0.05
Persistence of productivity shock	ρ_A	0.95
Standard deviation of productivity shock	σ_A	0.0044

Table 1 shows the parameterization of the economy based on second order approximation and no policy response. Parameters concerning the non-financial sector are standard in the literature and are borrowed from Dedola, Karadi and Lombardo (2013). I depart from the literature by choosing a steady state in which the financial constraints are slack[7][8]. The constraints may bind endogenously subject to adverse shocks. I define a financial crisis as the occasion in which the spread is two standard deviations above its mean. In the U.S., this definition corresponds to the early 20th century recession and the 2007 - 2008 financial crisis. The unconditional probability of a financial crisis is 5.28% under my calibration.

There are three parameters in the financial sectors. Following Gertler and Kiyotaki (2010), I choose a survival rate implying that averagely bankers survive for around 8 years. Next, I set the steady-state leverage ratio to 4. Given the financial constraint being slack in the steady state, this pins down $\varpi = \left(1 - \frac{1-\bar{r}_n}{\beta}\right)/\bar{\phi}$. The steady state proportion of divertable assets $\bar{\theta}$ is chosen such that OBC is close to zero in the steady state.

There are three exogenous variables in each country, namely productivity, A_t, capital quality, ξ_t, and fraction of divertable assets, θ_t. Each of them follows an uncorrelated AR(1) process. Parameters for the productivity are taken from the estimate of Heathcote and Perri (2002). Parameters for the capital quality follows Mark Gertler, Nobuhiro Kiyotaki and Albert Queralto (2012), the working paper version of which provides the microfoundations in the appendix. I chose a standard deviation of θ_t to make the mean of the annualized spread about 2.35%[9]. However, without features such as liquidity premia and true default risk, I inevitably overestimate the standard deviation of the spread. Or I would underestimate the mean if I calibrate the model to match the standard deviation.

As a robustness check, I also consider other relevant calibrations. For example, $\bar{r}_n$ is set to match a dividend rate of 5.15% made by the largest 20 U.S. banks during 1965–2013[10]. Quint and Rabanal (2017) use GMM to estimate a similar model with nominal friction, a Taylor rule, and an always binding financial constraint. They find a much large steady-state leverage being 16. This is probably not very surprising as Gertler and Kiyotaki (2010) consider the leverage being 4 as an average across sectors with vastly different financial structures. All the alternative calibrations change my results quantitatively but do not change the main conclusions. In appendix A, I report the behavior of the economy without interventions under my main calibration, which is quantitatively similar to the results of Dedola, Karadi and Lombardo (2013). However, I will show shortly how the OBC setting matters to the optimal policy.

[7]With integrated security markets and a steady state in which the financial constraints do not bind, there is indeterminacy between the bank's total assets, ω_t and ω_t^*. To pin down these variables, it is sufficient to introduce an asset adjustment cost to the bank's balance sheet:

$$\frac{\psi_\omega}{2}\left(\omega_t + AP_{1,t} - SteadyState\right)^2,$$

where $\psi_\omega = 10^{-5}$ in practice.

[8]This is sometimes impossible in other models, such as the one developed in Guerrieri and Iacoviello (2015).

[9]This figure is calculated from quarterly data of Moody's seasoned Bbb corporate bond yield relative to yield on 10-year treasury constant maturity, 1983q1-2017q1, retrieved from FRED, Federal Reserve Bank of St.. By contrast, Gertler and Kiyotaki (2010) and Dedola, Karadi and Lombardo (2013) use 1%, which is roughly the mean of Moody's seasoned Aaa corporate bond yield relative to yield on 10-year treasury constant maturity.

[10]This number is calculated by Swarbrick, Holden and Levine (2017) using Baron (2017)'s dataset

There are two policy-related parameters. I choose $\theta_g = 1$, meaning that discount window lending cannot be diverted. This choice makes the three policies more comparable because the funds of the other two policies are effectively non-divertable too. However, as noted by Gertler and Kiyotaki (2010), there is likely a capacity constraint on the central bank's ability to retrieve funds. Thus, the true θ_g could be smaller. Reducing this parameter in the interval $\theta_g \in (0, 1]$ makes discount window lending less effective, which in turn require stronger interventions. The second policy parameter is the intervention cost τ. Since it is difficult to measure in data the inefficiency of public activism in private financial markets, I set it to a number small enough to allow me focusing on the benefits of interventions. I consider larger intervention cost later.

IV. The Ramsey cooperative policy

In this section, I focus on a benchmark case in which the Home and Foreign governments conduct cooperative policy, and the policy is financed by government bonds (or equivalently a lump-sum tax) and entails only small reduced form cost.

The government in each country jointly maximizes a single objective function - the life-time utilities averaged across countries - by committing to a state-contingent plan of one of the three policies discussed early. Policy makers solve the following problem:

$$\min WEL_g \equiv$$
$$-E_t \sum_{j=t}^{\infty} \beta_{t,t+j}^g \frac{1}{2} \left[\frac{(c_{t+j} - hc_{t+j-1})^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{1+\varphi}}{1+\varphi} + \frac{\left(c_{t+j}^* - hc_{t+j-1}^*\right)^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{*1+\varphi}}{1+\varphi} \right]$$

subject to all the equilibrium conditions of private agents[11], where $\beta_{t,t+1}^g$ is identical to the discount factor of the households. In solving for the optimal policy I follow the "timeless" perspective advocated by Woodford (2003). However, the resulted system is very difficult to simulate especially when I employ the SEP alike solution. Thanks to the small policy cost, I can solve a slightly simplified problem with almost identical results to the true Ramsey problem. Specifically, I make the following assumption

ASSUMPTION 1: *The governments conduct policy subject $\lambda_t = \lambda_t^* = 0$ instead of $\lambda_t \geq 0$ and $\lambda_t^* \geq 0$.*

According to equation 10, the positive multiplier implies a positive spread. In the true Ramsey problem, the governments can tolerate positive spreads (at an order or magnitude 10^{-6} given my calibration) to the extent that the marginal benefits of reducing the spreads equal the marginal cost of intervention. Hence we have

[11]These conditions include two inequalities, $\lambda_t \geq 0$, $OBC_t \geq 0$, a slackness condition $\lambda_t OBC_t = 0$, and their Foreign counterparts. It can be verified that$\lambda_t \geq 0$ is a redundant constraint. Intuitively, $\lambda_t \geq 0$ roughly implies $r_{k,t+1} - r_t \geq 0$ according to equation 10, which a benevolent policy maker would never violate.

$\lim_{\tau\to 0^+}\lambda_t = \lim_{\tau\to 0^+}\lambda_t^* = 0$ in the Ramsey equilibrium, which justifies assumption 1 when τ is small. Results in this section remains unchanged if I do not make assumption 1.

A. *Impulse response analysis*

Following the literature, I consider policy responses to two shocks. First, I consider a negative capital quality shock ξ_t. The impacts of this shock can be decomposed into two stages. In the first stage, this shock has real impacts similar to those of a productivity shock. In particular, the return on securities is low, which reduces the banks' net worth by a multiplier of their leverage. Consequently, the financial constraints may bind and the second stage "financial accelerator" effects take place. In the case of binding, banks must firesell their assets. Since banks take asset prices as given, it is their externality that the firesale depresses asset prices and further impairs their net worth. As a result, the financial constraint binds even tighter. In the second stage, banks are inefficient financial intermediaries. The second shock λ_t , referred to as a financial shock, tightens the financial constraints directly (Perri and Quadrini, 2011; Dedola and Lombardo, 2012; Negro et al., 2017), and hence has only the second stage effects. How the economy responds to these shocks under no policy intervention can be found in appendix A.

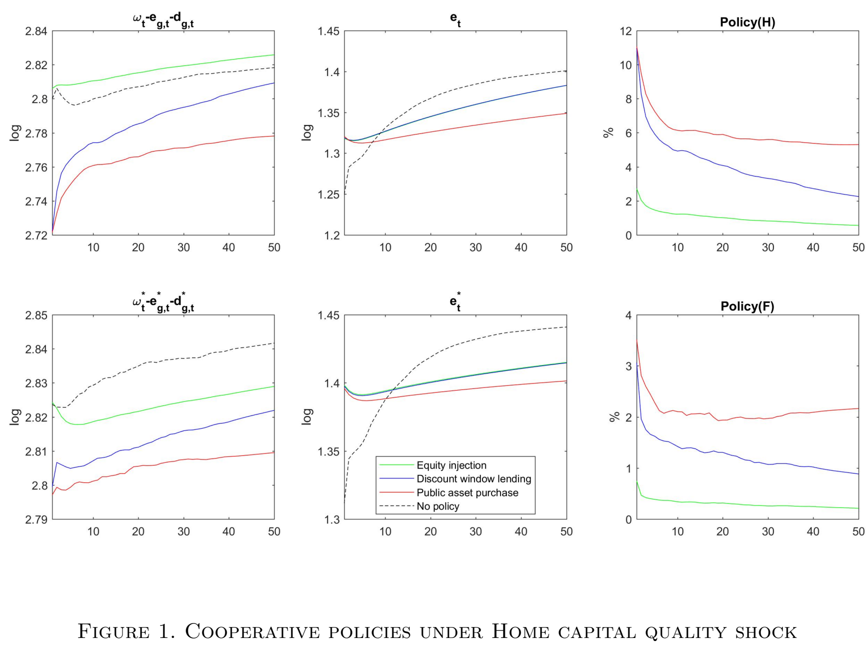


FIGURE 1. COOPERATIVE POLICIES UNDER HOME CAPITAL QUALITY SHOCK

Figures 1 and 2 plot the impulse responses to one standard deviation of the two shocks for three variables, namely banks' assets financed by private funds, private

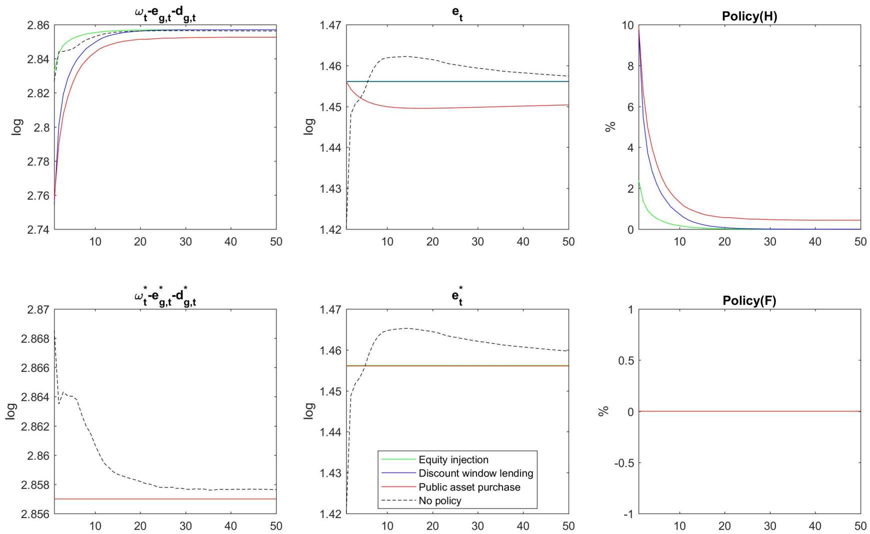


Figure 2. Cooperative policies under Home financial shock

equities, and the policy spending as a fraction to domestic asset value. Other variables are not shown because they behave as if there were no financial constraint, i.e., like the black broken lines in Figures A1 and A2 in the appendix. The results obtained with public asset purchases, discount window lending, equity injections, and no policy are shown in red, blue, green, and black, respectively.

By design, all unconventional polices address the financial constraints and hence only the second stage effects of the shocks. They share two common roles. First, policy injects public funds into the economy when banks are inefficient in intermediating private funds. The public funds support asset prices at their optimal level and hence undo any effect of the bank's externality. Consider for example public asset purchases. Banks can sell more of their assets and retain higher level of private equities (or net worth) than the no policy case. As a result, public funds effectively crowd out private funds. This crowding out effect implies that, however, private equities grow slowly. To stabilize the financial sector, the economy must also exit slowly from the interventions until banks accumulate enough private equities. The exit path needs to be consistent with the growth path of the private equities.

Since the economy must exit from interventions eventually and conducting the policy entails a non-zero cost, we would like to exit as soon as possible. The second role of policy is to relax the financial constraint 9, which reduces the crowding effect because banks can raise more deposits from households and rely less on public funds. Consequently, the exit from then interventions is faster. Since the government pays resource cost to intervene, the policy that requires the least interventions is the most efficient policy. It is clear from the figures that, under either shock, equity injections have almost no crowding effect and hence dominate the other two policies. How discount window lending can relax the financial constraints and hence its efficiency depend on θ_g. Given $\theta_g = 1$, discount window lending is the second efficient policy. But it can be less efficient than public assets purchases with a smaller θ_g (not shown). However, the effectiveness of public asset purchases could be underestimated because there is no friction between the banks and the non-financial firms. Such friction could potentially be addressed by the public asset purchases but not the other two policies.

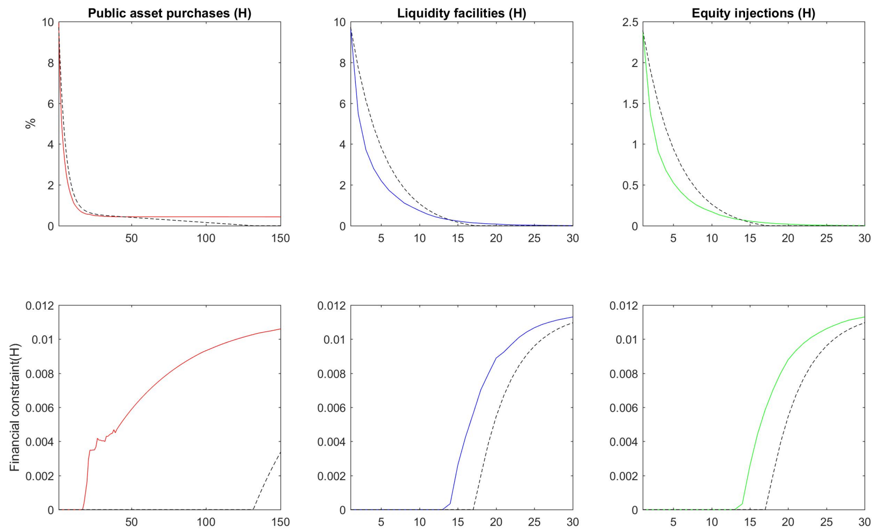
For all policies, their second role and their effectiveness can also be affected by the moral hazard as specified by equations 15 and/or 16. The moral hazard is arguably more likely to happen to equity injections. For example, the government may wish to increase the dividend rate $r_{n,t}$ for its own benefits, or the fraction of divertable funds θ_t may increase due to the public's concern of corruption. However, it is hard to form a prior for the moral hazard. Therefore, I don't draw a firm conclusion with respect to the policy ranking but emphasize that the policy efficiency lies on the second role of the policy.

On the international dimension, the policy responses are asymmetric. Following a Home capital quality shock, Foreign interventions are roughly half as strong as Home interventions. This is because banks hold a portfolio that consists of more domestic assets ($\alpha^p = 0.4$). Following a Home financial shock, however, Foreign needs not to intervene at all[12]. Since a financial shock does not have the first stage real impacts, Foreign banks would only be affected by depressed asset prices worldwide if there is no policy response. Home interventions fully stabilize asset prices in both country (the first role) so Foreign can enjoy a free ride.

B. *Precautionary effects*

The slow exit from interventions we see above is partially shaped by precautionary effects due to the future risk of binding financial constraints. In figure 3, I compare the EP alike solution (black broken lines) and the SEP alike solution (colored solid lines) to the impulse responses to a Home financial shock. The SEP alike solution is also employed in the previous subsection. If no risk of future bonding constraints is taken into account, unconventional policy ends in the same period when the economy escapes from the financial constraint. By contrast, if this risk is predicted by both the private agents and the government, the precautionary policy is relatively stable and persistent in the longer term while the economy has escaped from the constrained even sooner than the former case. Intuitively, the policy should give some precautionary protection to banks in a few periods after a crisis, during which the banks, though have enough net worth to escape from the financial constraints, are vulnerable to another adverse shock. The precautionary effects are particularly strong for public asset purchases such that the interventions are almost permanent even there is no unit root nor eigenvalues close to unit.

[12]Without assumption 1, foreign interventions approach to zero as the policy cost approaches to zero



Note: Colored solid lines are SEP alike results and black broken lines are EP alike results.

FIGURE 3. COOPERATIVE POLICIES WITH AND WITHOUT PRECAUTIONARY EFFECTS

V. The Ramsey noncooperative policy

Without international cooperation, everything else in the cooperative policy problem applies but each government now maximizes domestic welfare using domestic instruments, taking the entire path of foreign instruments as given. The equilibrium is an outcome of an open-loop dynamic Nash game. Following Coenen et al. (2007), the open-loop game is an unrealistic assumption but a necessary simplification to the problem. In simulating the model, it is important not to make assumption 1. The downside is that I can only simulate the model reasonably fast with the EP alike solution. I confine my discussion to the most efficient policy, equity injections. Other policies generate similar results.

As shown in figures 5 and 4, the noncooperative equilibrium is identical to the cooperative equilibrium when the unconventional policy is very cheap to use ($\tau = 0.0001$). Increase τ makes it favorable to share intervention cost across countries. Given a Home shock, this means fewer interventions by the Home government and more by the Foreign government, which results in higher credit spreads globally. However, after a certain point, increasing τ further affects the noncooperative policy more than the cooperative policy, and cooperation gain jumps from zero to a positive number. The jumping point is about $\tau = 0.01$ for a capital quality shock and about $\tau = 0.001$ for a financial shock. As a result, given a Home shock, the noncooperative equilibrium features excessive interventions in Foreign and insufficient interventions in Home. The effect of noncooperation on credit spreads depends also on the shock.

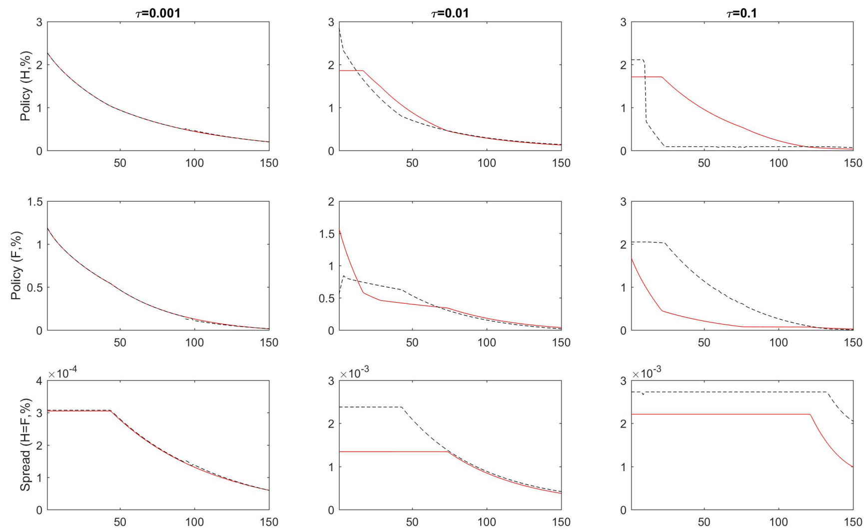


FIGURE 4. NONCOOPERATIVE POLICY RESPONSES TO A HOME CAPITAL QUALITY SHOCK

Under a capital quality shock, the spreads are higher and more persistent in the noncooperative equilibrium than in the cooperative equilibrium. Under a financial shock, however, the noncooperative policy achieves smaller spreads in the short run. My result differs from that of Dedola, Karadi and Lombardo (2013) in terms of the jump. In their paper, increasing the cost after a certain point makes the noncooperative policy jump to zero in both countries. Increasing the cost further makes even the cooperative policy jump to zero too. By contrast, in my context, only the difference between the cooperative and noncooperative policy jumps.

VI. Simple Rules

It is well known that the Ramsey solution is mute with regard to policy implementation. In this section, I examine the extent to which various simple feedback rules can approximate the Ramsey policy. I focus on the cooperative case. The noncooperative simple rule is studied by Dedola, Karadi and Lombardo (2013). The benchmark rule proposed in the literature is

$$\mathcal{P}_{i,t} = \kappa E_t \left(r_{k,t+1} - r_t \right) \tag{17}$$

in Home and similarly in Foreign where $\mathcal{P}_{i,t}$ is the policy spending proportional to domestic asset value, i indexes the three policies, parameter κ determines aggressiveness of the interventions. I refer to this rule as the spread rule. Foerster (2015) proposes an improvement over the benchmark rule by adding an autoregressive term

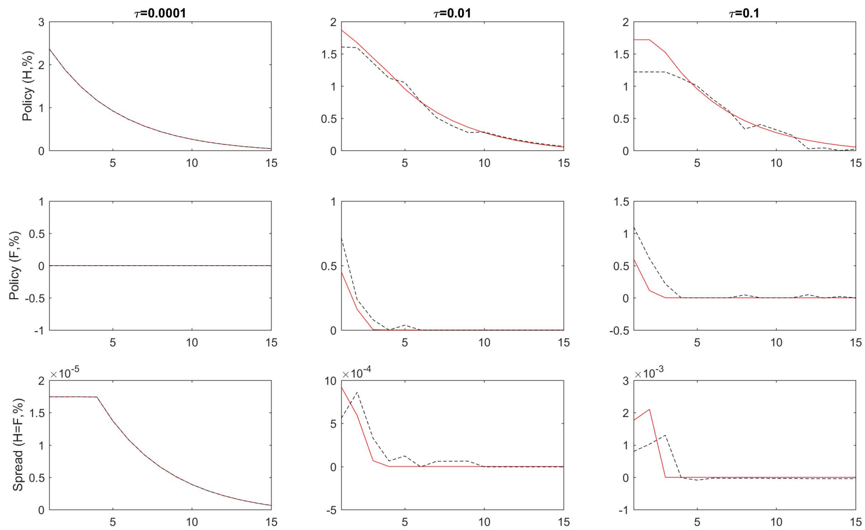


Figure 5. Noncooperative policy responses to a Home financial shock

(AR spread rule):

$$\mathcal{P}_{i,t} = \kappa \left(1 - \rho_{\mathcal{P}}\right) E_t \left(r_{k,t+1} - r_t\right) + \rho_{\mathcal{P}} \mathcal{P}_{i,t-1}. \tag{18}$$

Based on my discussion of the first role of the policy in section IV.A, another rule that is intuitively promising is to respond to asset price gaps (price rule)

$$\mathcal{P}_{i,t} = -\kappa \left(\ln q_t - \ln q_{t,potential}\right), \tag{19}$$

where I use potential variable to refer to the realization that would occur in a world without the financial constraints, and use gap variable to refer to the difference between the variable and its potential counterpart. The negative sign before κ reflects the fact that asset prices are low during a financial crisis.

Table 2—Unconditional welfare loss under simple rules

	Asset pur.	Dis. win.	Equity inj.
Spread rule	12.29 (150)	6.84 (150)	4.30 (115)
AR spread rule	12.22 (150, 0.9)	5.80 (150, 0.9)	4.18 (115, 0.9)
Price rule	6.28 (400)	3.83 (450)	2.49 (450)

Note: The numbers outside the brackets are base points of unconditional welfare loss under an optimized rule relative to that under the optimal allocation. The numbers inside the brackets are the parameters of the optimized rules.

The unconditional welfare loss achieved by the optimized version of these rules[13] are reported in table 2. Again, the government needs to pay reduced form cost with τ=0.01. First note that, regardless the rule, the most efficient policy is again equity injections, followed by discount window lending and public asset purchases. Second, regardless the policy, adding an autoregressive term always improves the spread rule, but both spread based rules always generate much larger welfare loss than the price rule does.

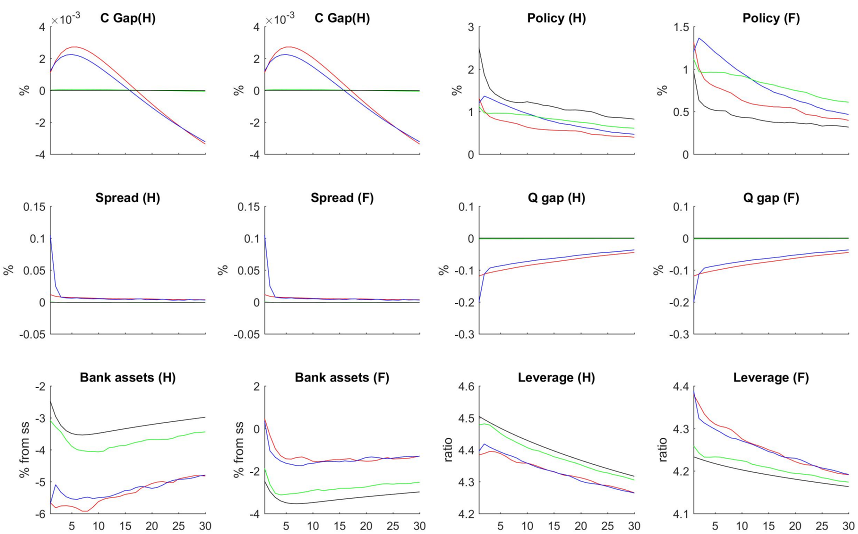


Figure 6. Response to Home capital quality shock, with different policy rules

Note: Gap variable is the difference between the actual variable that realizes in the model and its counterpart that would realize in a world without the financial constraints. C denotes consumption, Q denote the asset price.

To better understand how these rules are different, it is useful to consider impulse response of the economy under each rule. For the purpose of illustration, I only show the responses of equity injections in figures 6 and 7. Consider first the spread rule shown in red. Subject to either shock, the spread rule is not aggressive enough to stabilize the financial sector, living a significant gap in asset prices and spreads. As a result, there is also a substantial fluctuation in consumption gap. However, it exits from interventions at roughly the same speed as the Ramsey results. Adding

[13]The welfare loss achieved by the Ramsey policy relative to the optimal allocation is negligible. I also examine rules responding to the leverage gap, the leverage deviation from steady state, the change in spread, and rules responding to multiple variables. All of these render a larger welfare loss. In particular, responding to the variable's deviations from its steady state is usually a bad choice because the government intervenes even when the shocks are not strong enough to trigger a financial crisis, in which case the economy suffers an unnecessary intervention cost. In addition, the unnecessary crowding out effect makes banks vulnerable to further adverse shocks.

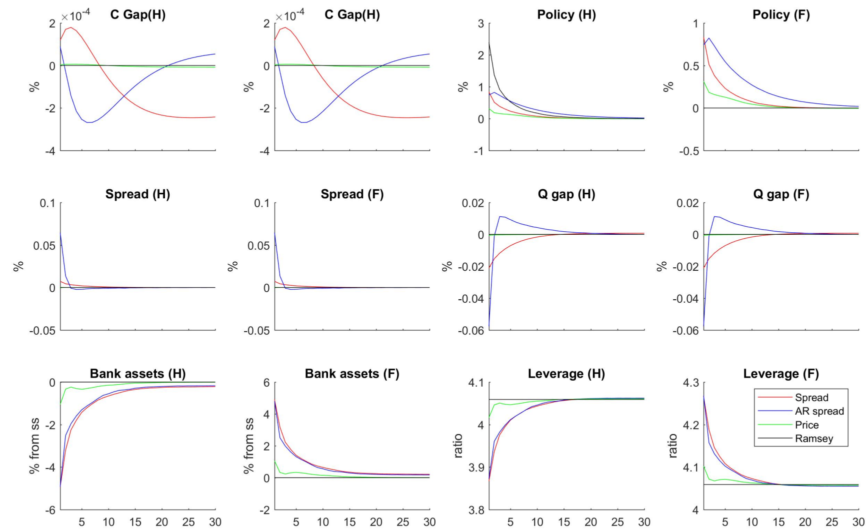


FIGURE 7. RESPONSE TO HOME FINANCIAL SHOCK, WITH DIFFERENT POLICY RULES

Note: Gap variable is the difference between the actual variable that realizes in the model and its counterpart that would realize in a world without the financial constraints. C denotes consumption, Q denote the asset price.

an AR term (shown in blue) make the interventions more persistent in response to a financial shock but less persistent in response to a capital quality shock. Generally the AR spread rule features stronger interventions then the naive spread rule. It allows relative high spreads and price gaps in the first few period but make them smaller thereafter. So the fluctuation in consumption is smaller and the life-time utility is improved. This improvement is particularly clear under the capital quality shock. Interestingly, the AR spread rule features hump-shaped responses to the shock, which seems to capture the observation that central banks tend to strengthen unconventional policy at the early stage of the crisis.

The price rule (shown in green) almost replicates the Ramsey outcomes. There is barely any fluctuation in the consumption gaps, the asset price gaps, and the spreads. This is expected because the main role of unconventional policy is to support asset prices. However, surprisingly, the price rule achieve these with even fewer interventions than the Ramsey policy under the financial shock. The only reason the price rule still generates substantial welfare loss is that the government can not customize rule parameters to different shocks. Consequently, facing a Home shock, interventions are relatively weak in Home but relatively strong in Foreign, and visa versa facing a Foreign shock. In addition, the price rule is not practicable subject to a capital quality shock or any other real shock because the potential asset price is not observable.

VII. Conclusions

I study the Ramsey optimal unconventional monetary policy in a two-country version of Gertler and Kiyotaki (2010). The main findings are threefold. First, I suggest that unconventional policy should be designed to address the financial constraint banks face. Second, after giving a strong initial response, the central bank should exit slowly from the policy even after a financial crisis has passed. Third, if cross-country policy cooperation is not imposed, the interventions are too strong in one country but too weak in the other. The cooperation gain is zero if the intervention cost is small. Increasing the intervention cost beyond a certain point lets the cooperation gain jump to a positive number. I also evaluate several simple feedback rules. I suggest that the policy should respond to asset prices facing a financial shock but respond to the credit spread with an autoregressive term facing other shocks.

Naturally, this paper is subject to several limitations. First, the financial accelerator due to binding financial constraints has relatively small impacts on the economy. As a result, the benefit of the unconventional policy is small and dominated by the distorting effects of a tax. On the other hand, it is a strong assumption that unconventional policy is fully financed by distorting tax, which leads to urgent demand for empirical evidence of the cost of unconventional policy. Second, an extra benefit of adopting the unconventional policy could be to restore the functioning of financial markets through which conventional monetary policy works. I left the optimal interaction between unconventional and conventional monetary policy to further research. Third, there is no friction between the banks and the non-financial firms. Such friction could potentially be addressed by the public asset purchases but not the other two policies. Therefore, the benefits of public asset purchases could be underestimated in this paper.

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Competitive Equilibrium

Here I examine quantitative behaviors of the model without policy interventions, and the size of precautionary effects that originate from the OBCs.

A1. Impulse response analysis

I consider responses to two shocks, namely a standard deviation negative Home capital quality shock ξ_t and a standard deviation positive Home financial shock θ_t. I use black broken lines to represent variables that would be realized in a financially frictionless world (potential variables) and red solid lines to represent actual variables.

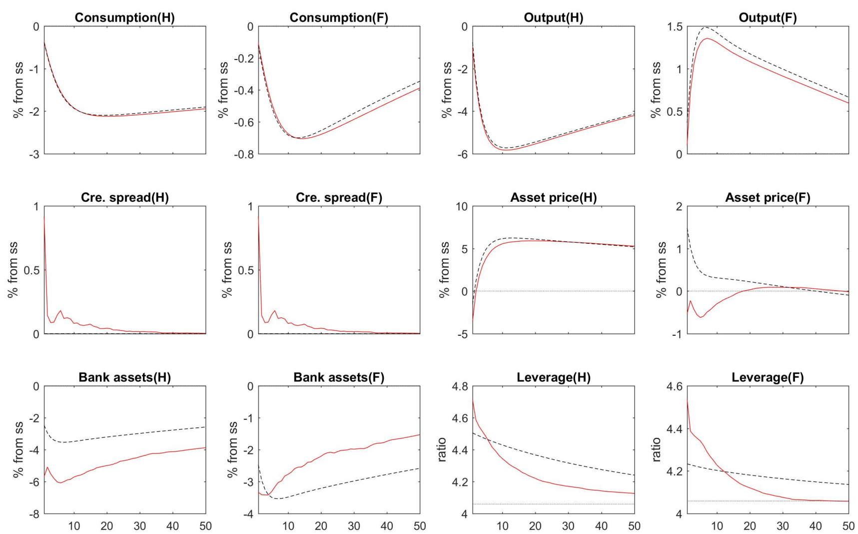

FIGURE A1. RESPONSE TO HOME CAPITAL QUALITY SHOCK

Note: Black broken lines represent variables that would be realized in a financially frictionless world (potential variables), red solid lines represent actual variables, and dotted line represents the steady state.

As shown in figure A1, the capital quality shock brings a deep and persistent global recession. Thanks to the OBC setting, I can decompose the effects of this shock into two "stages". In the first stage, the shock has a real impact shown by the black broken lines. When the financial constraints bind, there are second stage "financial accelerator" effects. In this case, banks must firesale their assets, which depresses asset prices and further impairs their net worth. As a result, the financial constraints bind even tighter. The positive spreads suggest that banks are inefficient financial intermediaries. Overall, the second stage effects amplify the first stage effects. The shock propagates to Foreign via the equalization of asset returns across countries, as suggested by equation 11, and a diversified portfolio. With a portfolio featuring home bias ($\alpha_p = 0.4$), Foreign banks suffer a smaller loss on their net worth than Home banks. Figure A2 shows that the financial shock has only the second stage impacts on the economy. Since all assets are healthy, Foreign

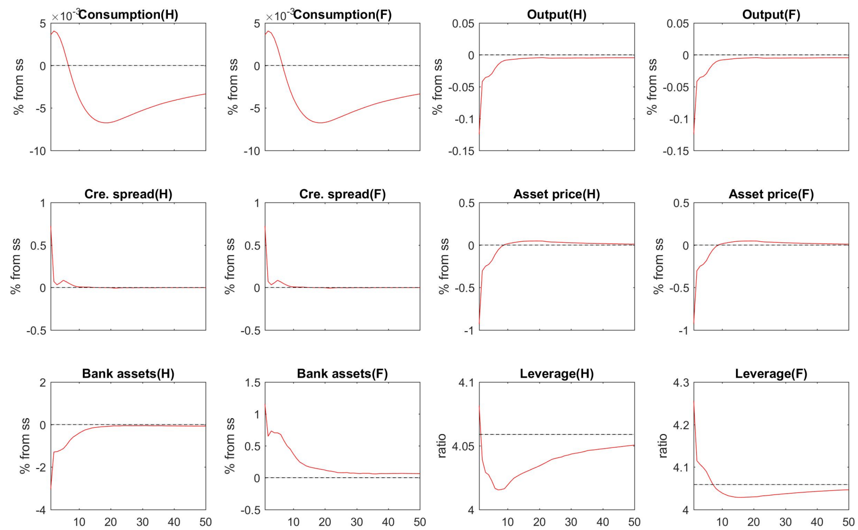

FIGURE A2. RESPONSE TO HOME FINANCIAL SHOCK

Note: Black broken lines represent variables that would be realized in a financially frictionless world (potential variables), red solid lines represent actual variables, and dotted line represents the steady state.

banks would like to increase their asset holding while Home banks are forced to firesale their assets. However, Foreign banks only partially compensate the drop of investment. The global investment decreases, which affects consumption and output in both country symmetrically.

No matter whether the financial constraints bind forever or occasionally, the second stage effects are much smaller than the first stage effects. The literature notes at least two reasons why this is the case. Jakab and Kumhof (2015) suggest that banks in the real world provide financing through money creation but banks in most models accept pre-existing real resources from savers and then lend them to borrowers. They find that adding money creation to the model allows the same shocks to have much greater effects on the non-financial sector. Negro et al. (2017) highlight a role of the nominal rigidity and the zero lower bound without which the financial friction accounts for a drop in investment, but not in output, thanks to a rise in consumption.

A2. The precautionary effects of risk

If the financial constraints are always binding, banks always hold the maximum level of assets permitted by their net worth. However, in the OBCs setting, the amount of assets held by banks is also affected by the likelihood that financial constraints are binding in the future. This is known as the precautionary effects of risk. To visualize the effects, I simulate the model with and without integrating over future uncertainty. The difference between the SEP alike solution and the EP alike solution shows the precautionary effects.

Table A1—Unconditional Mean (StD) of Home variables

Variables	$p\,(Crisis)$	Bank's assets	Annualized Spread	Consumption
EP alike	5.48%	4.72	2.89%	0.316
		(0.90)	(5.34)	(0.22367)
SEP alike	5.28%	4.47	2.33%	0.313
		(0.76)	(4.62)	(0.22375)

Table A1 compares the sample mean and the standard deviation of a few variables computed from the two simulations. First, the precautionary effects reduce the probability of a financial crisis (defined in the main text) by 0.2%. To avoid being constrained, banks would like to hold fewer assets on average. If banks do not do so, they suffer from a larger volatility of asset holding. The precautionary effects also reduce both the mean and standard deviation of spread. However, the precautionary effects are small on non-financial variables. In a smaller open economy model with occasionally binding collateral constraint, Mendoza (2010) also finds that long-run business cycle moments are largely unaffected by precautionary savings.

Fiscal distortion

While policy cost plays an important role in shaping the optimal unconventional policy, there is no hard evidence to quantify it in the reduced form. Another possible form of policy cost is a distorting effect of a tax by which the unconventional policy must be financed. To simulate the model in this case, again, I do not make assumption 1.

Suppose that the most efficient policy, i.e. equity injections, is solely financed by a sales tax, figure B1 plots the cooperative responses to a Home financial shock. The unconventional instruments are not shown because they are only used passively. The true instrument here is the sales subsidy, which is financed by a negative unconventional policy. This fiscal stimulation boosts investment and hence asset prices. Banks earn a fortune from their investment and escape from the financial constraints. In contrary to the unconventional policy, the fiscal policy addresses a financial crisis from the demand side of capital. However, the credit spreads are considerably large, suggesting inefficient financial markets. Since these results are obtained from maximizing welfare, varying the unconventional instrument actively must worsen welfare because the necessary sales tax to finance the active unconventional policy would have a strong distortionary effect on the economy. In other words, the distorting tax is a expensive source of fund to finance the unconventional policy. This result is robust to the nature of the shock, the chosen unconventional instrument, the

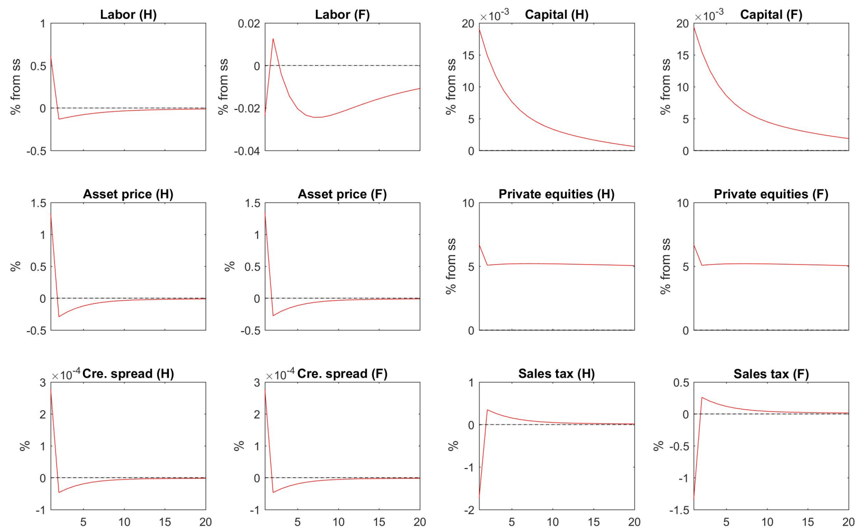

Figure B1. Cooperative fiscal policy under a Home financial shock

chosen fiscal instrument (such as a labor tax), alternative reasonable parameterization of the model such as a smaller Frisch elasticity of labor supply[14]. Nonetheless, this result is not very surprising because the benefits of the unconventional policy is bounded from above by the welfare losses caused by the second stage effect of a shock. In this and similar models, such losses are relatively small (see, Dedola, Karadi and Lombardo 2013, and appendix A) so the benefits of the unconventional policy are dominated by the distorting effect of the tax. If a smaller proportion of unconventional policy is financed by the tax, active unconventional policy becomes cheaper but active sales subsidy becomes more expensive. The threshold below which the unconventional policy is active is about 15% of the unconventional policy financed by the sales tax.

[14]Brendon, Paustian and Yates (2011) study a similar question where the tax follows a simple rule and the government budget is balanced by government debts. It is not surprising that the distorting tax plays a minor role in their paper.