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Inequality Adjustment Criteria for the Human Development Index

Enrico Casadio Tarabusi[1], Giulio Guarini[2]

Abstract

Despite its popularity, the United Nations' Human Development Index (HDI) only addresses simplistically, if at all, issues of inequality, intended either across dimensions or across units (or both). To overcome this problem, the weighted arithmetic average can be replaced, in the aggregation steps, by more sophisticated non-linear functions, often given by suitable generalised means, that impose penalizations for inequalities; this is done (more or less explicitly) in the literature, as well as in the 2010 edition of the Human Development Report (HDR). Besides other basic properties that aggregation functions are expected to satisfy, the following additional two appear relevant: the function must be defined for every set of values of variables (including high or negative), and the compensability among variables must be incomplete. Furthermore, a choice must be allowed among three different kinds of penalisations: one that only depends on the differences of variables (called "constant penalisation" here); one that, for given such differences, increases--and one that decreases--when the absolute levels of variables increase. These features were not discussed previously in the literature and are not fulfilled, for instance, by the Inequality Adjusted HDI of the 2010 HDR. Nevertheless, these features do hold for a suitable explicit generalised mean introduced here. Such an aggregation function is then applied to a database of 32 developing or developed countries, thereby resulting in significant rating and ranking variations with respect to the HDI, especially in the non-constant penalisation cases. Moreover, there is a negative correlation between the HDI and the penalisation value (that can be regarded as a penalization index in itself), both in terms of rating and ranking.

JEL : O15, D63
Keywords: inequality, human development index, aggregation functions

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We want very thank Marcella Corsi and Paolo Palazzi for the useful comments.

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Abstract

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Introduction

The Human Development Index of the United Nations is the most important index used in the development analysis. The index aggregates with arithmetic mean variables regarding three fundamental dimensions of development: income, education, health. Until 2010, one of the most relevant criticisms of this index has been the absence of inequality measure, even if the Human Development Report underlined the relevance of the distribution.

"Presenting average figures for each country disguises many important disparities – between urban and rural areas, between rich and poor, between male and female, as well as between ethnic groups and different regions. The HDI should try to reflect how people really live. (UNDP 1992 p. 21)"

Thus, in the Human Development Report 2010 there is a new Inequality Adjusted Human Development Index. Our goal is to build a new index of Human development by measuring the inequality aspects. Specifically, we use a new method of aggregation to take in to account the inequality among dimensions and among groups. In fact, it is necessary to consider inequality in all dimensions of human development. "The extent of real inequality of opportunities that people face cannot be readily deduced from the magnitude of inequality of *incomes*, since what we can or cannot do, can or cannot achieve, does not depend just on our incomes but also on the variety of physical and social characteristics that affect our lives and make us what we are." (Sen, 1992 p.98)

About inequality there is a distinction between horizontal and vertical inequality. "It is my hypothesis that an important factor that differentiates the violent from the peaceful [countries] is the existence of severe inequalities between culturally defined groups, which I shall define as horizontal inequalities to differentiate them from the normal definition of inequality which lines individuals or households up vertically and measures inequality over the range of individuals – I define the latter type of inequality as vertical inequality. Horizontal inequalities are multidimensional – with political, economic, and social elements (as indeed are vertical inequalities, but they are rarely measured in a multidimensional way). It is my contention that horizontal inequalities affect individual well-being and social stability in a serious way, and one that is different from the consequences of vertical inequality." (Stewart, 2003, p.3)

We survey some contributions (as UNDP 1993; Anand and Sen 1995; Hicks 1997; Foster et al. 2005; Stanton 2006; Grimm et al. 2008, 2010; Seth 2009; UNDP 2010) that try to integrate the Human Development Index with inequality measures and we show their criticisms about the mathematical aspects[2]. This is useful from the development point of view, because mathematical elements influence conceptions, theories and policies on development. Our index enjoys various basic properties of the Human Development Indices: continuity, symmetry in dimension, symmetry in people, replication invariance, positive monotonicity, linear homogeneity, normalisation, subgroup consistency, idempotence, stability for translations, progressive compensability, unrestricted domain, path independence, association increasing transfer. Moreover our Index enjoys other properties not enjoyed by the Inequality Adjusted Human Development Index of Human Development Report 2010: *unrestricted domain*, *incomplete compensability*, *level dependent inequality aversion*. According to this last property, our index penalises the inequality and it has a parameter according to which the index can be constant, decreasing, increasing for translation. That is, we can measure Human Development Index in three different ways according to theoretical assumptions we have regarding the relation between the level of human development and inequality. This point is very crucial because we want to specify all kinds of relations between the level of human development and inequality and correspondingly to point out different results. *Increasing penalisation* means that an unequal development at high development level is more serious than at low development level. On the contrary, *decreasing penalisation* means that for less developed countries the inequality is more serious. Finally *constant penalisation* means that the rate of inequality penalisation does not depend on the level of development. In this way, the method can be three laws of inequality penalisation, constant, decreasing, increasing with respect to the level of Human Development Index.

1. The Inequality Adjusted Human Development Indices

We want to survey some interesting examples of inequality adjustment of Human Development Index. Before starting we need to define some mathematical elements useful for the analysis. We define the matrix Q with *m* rows and *n* columns and q_{ij} its component on row *i* and column *j*, then we have

[2] For a general analysis of the composite indicators with an aggregation method with unbalance adjustment see Casadio Tarabusi and Guarini (2010).

$$Q = \begin{vmatrix} q_{11} & \dots & & q_{1n} \\ \dots & \dots & \dots & \dots \\ q_{i1} & \dots & q_{ij} & q_{in} \\ q_{m1} & \dots & q_{mj} & q_{mn} \end{vmatrix}$$

Thus, to simplify we define the row vector $q_i = (q_{11},...,q_{1n})$ and the column vector $q_j' = (q_{1j},...,q_{mj})$; with *i* we indicate the variable, while with *j* we indicate the individual (person, group, region, state…) of generic population. We define the following Inequality indices:

$I(q_i)$ that measures inequality within q_i, within-variable inequality;

$I(q_j')$ that measures inequality across q_j', across-variables inequality;

$I(Q)$ that measures inequality, within Q , that is across q_{ij}, overall inequality.

All indices we will present are built by using one type of the weighted generalised mean or quasi-arithmetic mean.

The *weighted generalised means* of Q, q_i and q_j are

$$\mu(Q) = f^{-1}\left(\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij} f(q_{ij})\right),\ \mu(q_i) = f^{-1}\left(\frac{\sum_{j=1}^{n} w_{ij} f(q_{ij})}{\sum_{j=1}^{n} w_{ij}}\right),\ \mu(q_j) = f^{-1}\left(\frac{\sum_{i=1}^{m} w_{ij} f(q_{ij})}{\sum_{i=1}^{m} w_{ij}}\right)$$

with $\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij} = 1$. Depending on the nature of $f(q_{ij})$ we obtain different types of weighted generalised means:

$\mu_{1-\varepsilon}(Q)$, the *power mean of order* $1-\varepsilon$, with $f(q_{ij}) = q_{ij}^{1-\varepsilon}$;

$\mu_a(Q)$, the *arithmetic mean*, with $f(q_{ij}) = q_{ij}$, that is when the order of power mean is 1;

$\mu_h(Q)$, the *harmonic mean*, with $f(q_{ij}) = q_{ij}^{-1}$, that is when the order of power mean is -1;

$\mu_g(Q)$, the *geometric mean*, with $f(q_j) = \ln(q_{ij})$.

The Human Development Index (*H*) uses only the weighted arithmetic mean $\mu_a(.)$, while the other indices can use also, or exclusively, the other types of mean, $\mu_{1-\varepsilon}(.)$, $\mu_h(.)$, $\mu_g(.)$. In fact, these means are used for the Inequality Adjusted Human Development Indices because, differently from $\mu_a(.)$, penalise the inequality (see Casadio Tarabusi and Guarini 2010).

Let us describe the original Human Development Index, *H*. We illustrate *H* by starting from the following selected variables:

- x_1 is the average life expectancy;
- x_2 is the weighted arithmetic mean of the adult literacy rate, with two-thirds weighting, and the combined primary, secondary, and tertiary gross enrolment ratio, with one-third weighting;
- x_3 is the income per capita calculated by the Purchasing Power Parity (US dollars).

We can present the building of *H* by showing three equations that represent the following steps: the transformation of variables (i), the normalisation of variables (ii) and the aggregation of variables (iii).

In the first step only the income per capita is transformed by the natural logarithm, then we have

(i) $f(x_1) = x_1, f(x_2) = x_2$ and $f(x_3) = \ln x_3$.

According to the Human Development Report the transformation of variables x_3 is caused by the assumption of the decreasing returns of the income per capita with respect to *H*. According to the UNDP (2005 p.341), "Income is adjusted because achieving a respectable level of human development does not require unlimited income. Accordingly, the logarithm of income is used." Using slightly different language, the first *H* explained the use of logarithms this way: "[Since] there are diminishing returns in the conversion of income into the fulfilment of human needs, the adjusted GDP per capita figures have been transformed into their logarithms." (UNDP 1990 p.13)

Subsequently, all variables are normalised by the min-max procedure represented by the following equation

(ii) $z_i = \dfrac{f(x_i) - \min[f(x_i)]}{\max[f(x_i)] - \min[f(x_i)]}$

with z_1, z_2, z_3 that are respectively the Health Index, Education Index, Income per capita Index. Moreover, $\min[f(x_i)]$ and $\max[(x_i)]$ are respectively the minimum and maximum function of x_i and their specific values are: 25 and 85 for Health Index, 0% and 100% for Education Index, ln(100$) and ln(40.000$) for Income per capita Index. Finally, one aggregates all Indices by the weighted arithmetic mean

(iii) $H = \mu_a(z_1, z_2 z_3)$.

After we present the procedures to build specific Adjusted Indices without considering that in some cases the variables x_1, x_2, x_3 can be proxies of the original ones because of the database problem, but this is not relevant for our analysis.

In 1993, the Human Development Report proposed an Inequality Adjusted Human Development Index (H_A). The aim of this new index is to correct the *H* by introducing the Gini Index that is a measure of the income inequality. This Report argues that the income distribution has a strong impact in the human development. The H_A differs from the *H* on transformation variable step (ii). The procedure is the following:

(1.i) $f_A(x_1) = x_1$, $f_A(x_2) = x_2$ and $f_A(x_3) = [1 - I_A(x_3)]\ln(x_3)$

(1.ii) $z_{Ai} = \dfrac{f_A(x_i) - \min[f_A(x_i)]}{\max[f_A(x_i)] - \min[f_A(x_i)]}$

(1.iii) $H_A = \mu_a\{\mu_a(z_{A1}), \mu_a(z_{A2}), \mu_a(z_{A3})\}$.

In this case *j* indicates person. In equation (1.i), the index $0 \le I_A(x_3) \le 1$ is the Gini Index; $I_A(x_3) = 0$ means equal distribution, while $I_A(x_3) = 1$ means maximum inequality. In general, the Gini Index of generic variable y is $g(y_i) = 1 - \frac{2}{n}\sum_{k=1}^{n-1} s_k$ with $s_k = \sum_{l=1}^{k} y_l \Big/ \sum_{l=1}^{n} y_l$ and $y_1 \le y_2 \le ... y_n$.

Anand and Sen (1995) propose an inequality adjustment of *H* by introducing a measure of gender inequality. This integration of the original index has relevant consequences for the theoretical and political issues. "There is considerable evidence of anti-female bias in some countries in the world. This takes the form of unequal treatment, to food, health care, education, employment, and income-earning opportunities, - and is reflected in differential achievements of women relative to men. […] We should like to use the HDI to illuminate the gender disparities that result from such unequal treatment" (Anand and Sen, 1995, p.11)

The steps of the Gender Related Development Index are

(2.i) $f_B(x_1) = x_1$, $f_B(x_2) = x_2$ and $f_B(x_3) = \ln(x_3)$

(2.ii) $z_{Bij} = \left(\frac{f_B(x_i) - \min[f_B(x_i)]}{\max[f_B(x_i)] - \min[f_B(x_i)]} \right)_j$

(2.iii) $H_B = \mu_a \{\mu_h(z_{B1}), \mu_h(z_{B2}), \mu_h(z_{B3})\}$.

In this example *j* represents the gender groups (male, female) and the penalisation of inequality is made by the harmonic mean, $\mu_h(.)$. In equation (2.ii), differently from original *H*, the $\min[f_B(x_1)]$ and $\max[f_B(x_2)]$ are respectively, 27.5 and 87.5 for females and 22.5 and 82.5 for males.

Hicks (1997) proposes another Inequality Adjusted Human Development Index by using the Gini Index of the all variables z_1, z_2, z_3. Hicks underlines to extend the inequality matters on all dimensions of human development because "[…] inequality is shown to be a problem not just in income, where it is arguably most severe, but in education and health, where inequalities are perhaps seen as more troubling. […] Further, measures of inequality of income have been criticised because income-based poverty is sometimes only temporary; income-inequality measures such as Gini coefficient do not directly address the permanence of the distribution. Educational and health /longevity inequality are more permanent phenomena and thus are more clearly of social import, and even moral concern" (Hicks 1997, p. 1294).

This new indicator differs from the original *H* on the aggregation function. The procedure is

(3.i) $f_C(x_1) = x_1$, $f_C(x_2) = x_2$ and $f_C(x_3) = \ln x_3$

(3.ii) $z_{Ci} = \dfrac{f_C(x_i) - \min[f_C(x_i)]}{\max[f_C(x_i)] - \min[f_C(x_i)]}$

(3.iii) $H_C = \mu_a \{\mu_a(z_{C1})[1 - I_C(z_{C1})], \mu_a(z_{C2})[1 - I_C(z_{C2})], \mu_a(z_{C3})[1 - I_C(z_{C3})]\} = H(1 - I_C)$,

where the last equality defines I_C. In this case *j* indicates person. The equation (3.iii) reproduces the Sen welfare standard that is based on Gini Index (Sen 1997).

Foster et al. (2005) propose another inequality adjustment of the Human Development Index (H_D) where it is taken into account inequality across individuals by introducing a parameter measuring the *inequality aversion* given the level of the Human Development Index.

"One of the main limitations of the HDI is that, by not including a distributional dimension, it is possible to have a country with a higher HDI than another, but where poverty is widespread or where large groups are left out of the development process. It is also possible to have improvements in the HDI simultaneously with stagnation or even deterioration in development for vast sectors of the population" (Foster et al. 2005, p.25).

In this case, the step that changes is the aggregation step (iii). The procedure is the following

(4.i) $f_D(x_1) = x_1$, $f_D(x_2) = x_2$ and $f_D(x_3) = \ln x_3$

(4.ii) $z_{Di} = \dfrac{f_D(x_i) - \min[f_D(x_i)]}{\max[f_D(x_i)] - \min[f_D(x_i)]}$

(4.iii) $H_D = \mu_a \{\mu_{1-\varepsilon}(z_{D1}), \mu_{1-\varepsilon}(z_{D2}), \mu_{1-\varepsilon}(z_{D3})\} = H(1 - I_D)$.

In this case *j* indicates region. Equation (4.iii) is the mean of order (1-ε), where I_ε is the Atkinson Index (1970) that measures the inequality. In fact, the coefficient $\varepsilon \geq 0$ measures the aversion to inequality: with $\varepsilon = 0$ equation (4.iii) becomes a weighted arithmetic mean, that is $H_D = H$ and in this case the aversion to the inequality is zero because there is not an adjustment that penalise the inequality within H.

Stanton (2006) proposes another kind of adjustment by using the Gini Index (H_E). The H_E differs from the *H* on step (i) and (iii). The main different from others is to consider logarithmic transformation for all dimensions and not only for the income as in the original *H*. This change means that health and education, as income, show decreasing returns in terms of human development. To explain this assumption, the author reports two quotes. The first one is about health: "The components of HDI, namely, life expectancy and educational attainment, are 'functionings' in the Sen's sense but their relative values need not be the same across individuals, countries, and socioeconomic groups. Besides, the 'intrinsic' value of a single 'functioning', namely, ability to live a health life, is not captured by its linear deprivation measure in HDI, since a unit decrease in the deprivation in life expectancy at an initial expectancy of 40 years is not commensurate with the same unit decrease at 60 years" (Srinivasan 1994, p.240). The second quote is about education: "[T]he early 'units' of educational attainments to a country should be of much higher value than the last ones. In the context of policy-making in a country with 30% adult literacy, improvements in literacy are of far greater urgency than the same for a country with 90% adult literacy." (Noorbakhsh, 1997, p.519)

The procedure is the following

(5.i) $f_E(x_i) = \sum_{j=1}^{n} \frac{1}{n}(\ln x_j)$

(5.ii) $z_{Ei} = \dfrac{f_E(x_i) - \min[f_E(x_i)]}{\max[f_E(x_i)] - \min[f_E(x_i)]}$

(5.iii) $H_E = \mu_a\{\mu_a(z_{E1})[1 - I_E(z_{E1})], \mu_a(z_{E2})[1 - I_E(z_{E2})], \mu_a(z_{E3})[1 - I_E(z_{E3})]\} = H(1 - I_E)$.

In equation (5.i) *j* represents *j*-individual or *j*-group. In this way, all variables are decreasing returns with respect to the final index. Moreover, the aggregation function in equation (5.iii) is similar to which of H_C in equation (3.iii).

Grimm et al. (2008; 2010) calculate the Income Quintile Human Development Index by these steps

(6.i) $f_G(x_1) = x_1, f_G(x_2) = x_2$ and $f_G(x_3) = \ln x_3$

(6.ii) $z_{Gi} = \dfrac{f_G(x_i) - \min[f_G(x_i)]}{\max[f_G(x_i)] - \min[f_G(x_i)]}$

(6.iii) $H_{G\,j} = \mu_a(z_{G1j}, z_{G2j}, z_{G3j})$.

Thus *j* indicates income quintile. In this case there is not an adjustment but the inequality is measured by the ratio between the lower and the upper income quintile. "The results showed the across all countries inequality in human development was very high, was typically larger in developing countries, and particularly sizable in Africa. This was not only due to an unequal income distribution, but also to substantial inequalities in education and life expectancy. In some middle income developing countries the richest quintile ranked among the high human development countries, whereas the poorest quintile ranked among the low human development countries". (Grimm et al. 2008 p.2)

Moreover we present the Index built by Seth (2009) with these steps

(7.i) $f_L(x_1) = x_1, f_L(x_2) = x_2$ and $f_L(x_3) = \ln x_3$

(7.ii) $z_{Li} = \dfrac{f_L(x_i) - \min[f_L(x_i)]}{\max[f_L(x_i)] - \min[f_L(x_i)]}$

(7.iii) $H_L = \mu_{1-\alpha}(\mu_{1-\beta}(z_{L11}, z_{L21}, z_{L31}),..., \mu_{1-\beta}(z_{L1j}, z_{L2j}, z_{L3j}),..., \mu_{1-\beta}(z_{L1n}, z_{L2n}, z_{L3n}) = H(1 - I_L)$,

where "parameter β can be interpreted as the parameter measuring the degree of substitution among dimensions; whereas parameter a can be interpreted as the inequality aversion parameter." (Seth 2009, p.387). The author wants to introduce the forms of the inequality in multidimensional context: the *distribution sensitive inequality* and the *association sensitive inequality*. The former concerns the distribution of the single dimension; the latter refers to the correlation among dimensions. "Importance of the first form of multidimensional inequality can be traced back to the importance of the single-dimensional inequality concerning the dispersion of the distribution. The second form of multidimensional inequality is important for two reasons. First, the various components of human development are synergistically related to one another. When all dimensions are strongly correlated, then higher achievement in one dimension strongly enforces higher achievements in other dimensions and any one dimension is sufficient for measuring human development. Conversely, less correlation among dimensions makes multidimensional analysis more informative. Therefore, the degree of association among dimensions clearly has relevance for multidimensional evaluations of human development. Secondly, the association-sensitive inequality is important from the point of view of policy recommendation." (Seth 2009, p.376)

Finally we present the Inequality Adjusted Human Development Index of Human Development Report 2010, built with these steps

(8.i) $f_M(x_1) = x_1$, $f_M(x_2) = x_2$ and $f_M(x_3) = \ln x_3$

(8.ii) $z_{M\,i} = \dfrac{f_M(x_i) - \min[f_M(x_i)]}{\max[f_M(x_i)] - \min[f_M(x_i)]}$

(8.iii) $H_M = \mu_g(\mu_g(z_{L11}, z_{L21}, z_{L31}),..., \mu_g(z_{L1j}, z_{L2j}, z_{L3j}),..., \mu_g(z_{L1n}, z_{L2n}, z_{L3n}) = H(1 - I_M)$,

"The IHDI [Inequality Adjusted Human Development Index] takes into account not only a country's average human development, as measured by health, education and income indicators, but also how it is distributed. We can think of each individual in a society as having a "personal HDI [Human Development Index]." (UNDP, 2010 p.87). About the relation between this Inequality Adjusted Human Development Index and the original Human Development Index, the Report affirms: "The IHDI will be equal to the HDI when there is no inequality across people, but falls further below the HDI as inequality rises. In this sense, the HDI can be viewed as an index of "potential" human development (or the maximum IHDI that could be achieved if there were no inequality), while the IHDI is the actual level of human development (accounting for inequality). The difference between the HDI and the IHDI measures the "loss" in potential human development due to inequality" (UNDP, 2010 p.87). For more details see Alkire 2010.

2. The Inequality Adjusted Exponential Mean (IAEM) function for the human development

We concentrate our analysis in the step (iii) regarding the aggregation function F(Z). We want to present a new Inequality Adjusted Human Development Index by a new aggregation function that we call Inequality Adjusted Exponential Mean (IAEM). We want the aggregation function of H_H F(Z) to enjoy following properties, given two matrices (*n* x *m)* Q and V.

Let us to indicate the basic properties for human development indices (See Foster et al. 2005 for more details).

<u>Property (i): *continuity.*</u> The function *F* is continuous on its domain.
<u>Property (ii): *symmetry in dimension.*</u> $F(Z) = F(V)$ with $V = SZ$ where *S* is a permutation matrix (each column and each raw have one 1 and the rest 0). Each variable in the Index has the same importance, in other words $w_{i\,j}$ does not depend on i.
<u>Property (iii): *symmetry in people.*</u> $F(Z) = F(V)$ with $V = ZS$ where *S* is a permutation matrix. Each individual in the Index has the same importance, in other words $w_{i\,j}$ does not depend on j.
<u>Property (iv): *replication invariance.*</u> $F(Z) = F(V)$ with $V = (Z,..,Z)$ (*k* times) with $k \geq 2$. Thanks to this property it is possible to compare to Index that regards population with different sizes.
<u>Property (v): *positive monotonicity*</u>. $F(Z) \geq F(V)$ with $v_{ij} \geq z_{ij}$. The index is increasing with the increase of each component.
<u>Property (vi): *linear homogeneity.*</u> $F(V) = \alpha F(Z)$ with $V = \alpha Z$ and $\alpha > 0$. In this case the increase of Index is proportional to the increase of individual.
<u>Property (vii): *normalisation.*</u> If zij =beta (with $\beta > 0$) for every i and j then $F(Z) = \beta$. In this case the value of the Index is the same to the value of each individual.
<u>Property (viii): *subgroup consistency*.</u> $F(V) > F(Z)$ with $F(V') = F(Z')$ and $F(V'') > F(Z'')$ where V', V'', Z', Z'' are the partitions respectively of matrices *V* and *Z*. This means that the Index is increasing to the increase of one subgroup of individual when the Index of all others does not change.

<u>Property (ix): *idempotence.*</u> $F(z_1,...z_n) = z_1$ if $z_1 = ... = z_n$.

Property (x): *stability for translations.* $F(z_1+\lambda,...z_n+\lambda)=F(z_1,...z_n)+\lambda$.

Property (xi): *progressive compensability.* For any two variables z_i, z_j and any point z, the rate of compensation between z_i and z_j is increasing if the variable z_j is increasing, with the constraint that index and the remaining variables are kept constant.

Property (xii): *path independence.* It requires that there are the same results both if the aggregation occurs first across individuals and then across dimensions, and if the aggregation occurs first across dimensions and after across individuals. $F(Z)=F(F(z_i))=F(F(z_j))$. The results are indifferent to the aggregation order.

Finally, we consider the *association-sensitive properties* proposed by Seth (2009). Before explaining the properties, it is necessary to define the "association increasing transfer": it occurs when matrix Q (3 x n), obtained from matrix V (3 x *n*) (different from Q), has the following components $q_{11}=\min(v_{11},v_{12}), q_{21}=\min(v_{21},v_{22}), q_{31}=\min(v_{31},v_{32})$ and $q_{12}=\max(v_{11},v_{12})$, $q_{22}=\max(v_{21},v_{22})$ $q_{32}=\max(v_{31},v_{32})$ and $q_n'=v_n'$ for all $n\neq 1,2$.

Property (xiii.a): *strictly decreasing under increasing association (SDIA)*. This axiom requires that if Q is obtained from *V* by an association increasing transfer thus $H(Q)<H(V)$; the weak version called WDIA is $H(Q)\leq H(V)$.

Property (xiii.b): *Strictly increasing under increasing association (SIIA)*. This axiom requires that if Q is obtained from *V* by an association increasing transfer thus $H(Q)>H(V)$; the weak version called WIIA is $H(Q)\geq H(V)$.

If the Index enjoys path independence does not enjoy properties SDIA or SIIA.

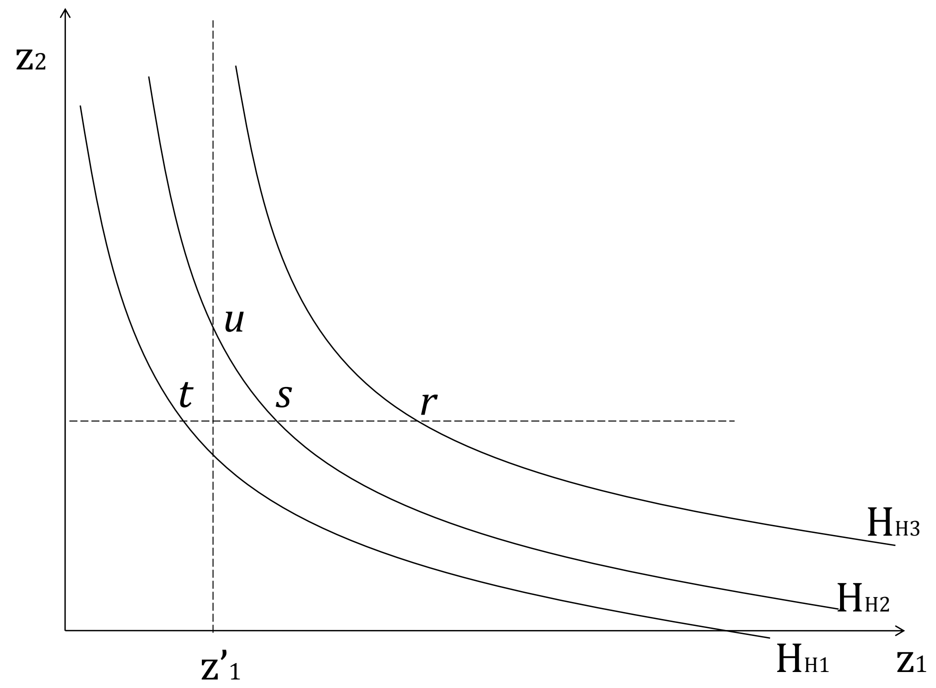
As Foster's function and Seth's function, IAEM function is can written as is can be $H_H=H(1-I_H)$ and the Atkinson's Index I_H enjoys the following properties: symmetry, replicant invariance and transfer principle. The *Pigou–Dalton transfer principle* (see Selth 2009) means that $I(Z^*)<I(Z)$ with $z_{kl}<z_{rs}$, $(z_{kl}*-z_{kl})=-(z_{rs}*-z_{rs})<0$. Thanks to this property, the transfer of income from a rich rises the Index.

We introduce three new properties that are not analysed by the literature regarding the functions that penalise inequalities and are not enjoyed by the Inequality Adjusted Human Development Index of Human Development Report 2010.

Property (xiv): *unrestricted domain.* The function *F* is defined on R^n. This is relevant because it is possible to use all normalisation, that is, one can use the most opportune normalisation according to the features of the analysis. The functions analysed before, that penalise inequalities, do not enjoy this property. For example, in the case of Inequality Adjusted Human Development Index built in the Human Development Report 2010 by using the geometric mean, the authors have to solve the problem of the zero and negative values that are not valid in the geometric mean: "The geometric mean in equation 1 does not allow zero values. For mean years of schooling one year is added to all valid observations to compute the inequality. Income per capita outliers—extremely high incomes as well as negative and zero incomes—were dealt with by truncating the top 0.5 percentile of the distribution to reduce the influence of extremely high incomes and by replacing the negative and zero incomes with the minimum value of the bottom 0.5 percentile of the distribution of positive incomes." (UNDP, 2010 p.218)

Property (xv): *incomplete compensability.* For every *j* and every given z_j' the set of values $F(z)$ with $z_j = z_j'$ has a finite upper bound. The functions analysed before, that penalise inequalities, enjoy this property, with the exception of the function $\mu_{1-\varepsilon}(.)$ with $0 < 1-\varepsilon \leq 1$.

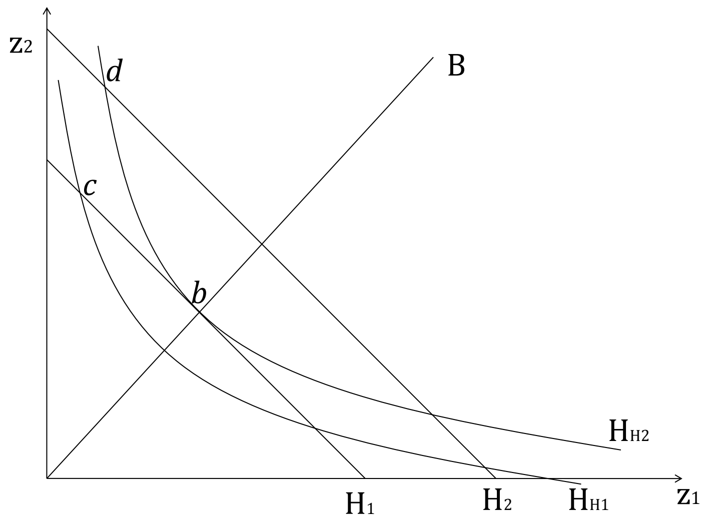
Figure 1



In other words, with complete compensability any decrease of any single variable can be compensated by suitable increases of the remaining variables; while in the case of incomplete compensability, only decreases in one single variable that are smaller than a given amount are compensable with suitable increases of the remaining variables. In Figure 1 the decrease of z_1 from point *s* to point *t* is compensable by the increase of z_2 from point *t* to point *u* (indeed $F(u) = H_{H2} = F(s)$; nevertheless, the decrease of z_1 from point *r* to point t cannot be compensated by any increase of z_2 from point *t*, because all the values $F(z_1', z_2)$ (where z'_1 is the first coordinate of *t*) are smaller than H_{H3}. Therefore, under incomplete compensability, starting from a given *n*-tuple of variables, for each single variable exists an upper bound of its decrease beyond which the same index value cannot be restored by increases in the other variables.

To specify the last property we introduce the concept of inequality penalisation by using Figure 2.

Figure 2



These parallel lines (or hyperplanes, for general m) are the level sets of the *H* Index and the parallel curves (or hypersurfaces) are the level sets of the H_H Index. We assume that the gradient of *F* on the balance line *B* is constant up to a proportionality factor. This assumption means that the tangent lines (or hyperplanes) to each level set of *H* at its unique balance point (given by the intersection of that level set with the balance locus B) are parallel; two of them are H_1 and H_2 in Figure 2. In order to obtain the value of penalisation of the H_H at a given unbalanced point *c*, say with z_2 strictly greater than z_1, take the unique balanced point *b* whose *H* value is *H*(c) (graphically, among those parallel lines take the only one that contains *c* and determine its intersection *b* with the line B); the requested penalisation is the difference of the values H_{H2} (the level containing *b*) and H_{H1} (the level containing *c*). Thus penalisation is

$P(c) = H_H(b) - H_H(c)$ that is $P(c) = H(c) - H_H(c)$

(the latter if property (ix) holds). This difference depends of course on the curvature of the level curves of H_H, but also on the steepness of H_H. This latter feature translates graphically into how many level curves for given equally-stepped values intersect a given segment parallel to the balance line.

Thus we can indicate the last property of IAEM function that concerns the inequality penalisation. The functions analysed before do not enjoy this property.

Property (xvi): *level dependent inequality aversion.* The inequality aversion changes with the level of human development given the amount of inequality. For every Z and whenever $\Lambda = (\lambda, \lambda, ..., \lambda)$ with $\lambda > 0$, if $P(Z + \Lambda) = P(Z)$ we have constant penalisation, if $P(Z + \Lambda) < P(Z)$ we have decreasing penalisation and if $P(Z + \Lambda) > P(Z)$ we have increasing penalisation.

According to the value of a parameter the function can have one of these kinds of law of inequality penalisation. This point is very crucial because we want to specify all kinds of relations between inequality and human development level and correspondingly to point out different results. *Increasing penalisation* means that an unequal development at high development level is more serious than at low development level. On the contrary, *decreasing penalisation* means that for less developed countries the inequality is more serious. Finally *constant penalisation* means that the rate of inequality penalisation does not depend on the level of development. In this way, the method can be three laws of inequality penalisation, constant, decreasing, increasing with respect to the level of Human Development Index.

The IAEM function is a modified exponential mean that is a special case of generalised mean $\mu(Q) = f^{-1}\left(\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij} f(q_{ij})\right)$ when $f(q_{ij}) = \exp(q_{ij})$. In fact, the Inequality Adjusted Human Development built by IAEM aggregation function is calculated in this way

$$H_H = F = f^{-1}\left(\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij} f(z_{ij})\right), \text{ where}$$

$$f(z) = \exp(-\varepsilon z - \gamma \exp(\gamma \varepsilon z)), \text{ and consequently}$$

$$f^{-1}(s) = -\frac{1}{\varepsilon}\left(\ln s + \frac{1}{\gamma} W\left(\frac{\gamma^2}{s^\gamma}\right)\right).$$

W is the Lambert function, also called Omega or Product-Log. An excellent approximation (less than 2‰ on the whole positive real halfline) of the Lambert function, found by Barry, Parlange, Lia, Prommer, Cunningham, Stagnitti (2000), is a function λ that only involves elementary functions, and can therefore be implemented in any spreadsheet for the calculation of the index. Explicitly:

$$\lambda(z) = (1+r)\theta(z) - r\varphi(z)$$

where $\theta(z) = \ln((z'/2)/\ln(z'/\ln(1+z')))$, $\varphi(z) = \ln(z''/\ln(1+z''))$, and $r = (\theta(e)-1)/(\varphi(e)-\theta(e))$, $z' = (12/5)x$, $z'' = (2z)$.

In the IAEM aggregation function parameter $\varepsilon > 0$ represents the Atkinson's inequality aversion, while the parameter γ represents the *level dependent inequality aversion*. Specifically for $\gamma = 0$ we have constant penalisation, for $\gamma = 1$ (or more generally for $\gamma > 0$) we have decreasing penalisation, and for $\gamma = -1$ (or more generally for $\gamma < 0$) we have increasing penalisation.

3. An application with income quintiles

We apply the IAEM function to the database of Grimm et al. (2010). The countries considered are 32 and represent all over the world: ten African countries (Burkina Faso, Cameroon, Cote d'Ivoire, Guinea, Madagascar, Mozambique, South-Africa, Zambia, Ghana, Ethiopia); seven Latin American countries (Bolivia, Colombia, Nicaragua, Brazil, Guatemala, Paraguay, Peru); four Asian countries (Indonesia, Vietnam, Kyrgyz Republic, India); ten European countries (Finland, Australia, Canada, France, Germany, Italy, The Netherlands, Poland, Spain, and Sweden) and the United States of America. The database concerns the Income Index (Z_Y), Education Index (Z_E) and Health Index (Z_H) calculated by income quintiles from variables obtained by different surveys (see Table A.5). For information on which variables are used for building indices see Grimm et al. 2010.

We make two different analysis characterised by same procedure. We consider three cases of inequality penalisation: decreasing (γ = 1), constant (γ = 0) and increasing (γ = -1). Moreover, we compare the index built by IAEM function (*H**) to the index built by arithmetic mean as in the original Human Development Index (*H*), in terms of ranking. Further we calculate the inequality penalisation index in terms of rating and ranking (*P*).

Finally, we calculate the ranking and rating correlation between the Index built by arithmetic mean *H* and the Inequality Penalisation Index *P*.

In the first analysis, we calculate an Adjusted Inequality Human Development Index by IAEM function where parameter ε assumes values according the Seth (2009) analysis, that is in the first aggregation across dimensions we have $\varepsilon = 1$ and in the second aggregation across income quintiles we have $\varepsilon = 3$ (see Table 1). Table 1 shows interesting results. With no constant penalisation the percentage of cases with rank difference (*H* - *H**) is very high: with decreasing penalisation is about 56 per cent, and with increasing penalisation is about 91 per cent. The highest rank difference regards the increasing penalisation. Two countries, that show this difference, are Mozambique (27) and Ethiopia (25) (see Table A.1). Moreover is negative the correlation between the original Human Development Index and the Inequality Penalisation Index both in terms of rating and in terms of ranking. In particular, the case of increasing penalisation has the highest correlation in terms of rating (-0.54), while the lowest rank correlation regards decreasing penalisation (-0.49).

Table 1

	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$
Percentage of cases with rank difference (H-H*)	56.25	18.75	90.63
Rating Correlation between H and P	-0.26	-0.50	-0.54
Spearman's Rank Correlation between H and P	-0.49	-0.65	-0.65

H: Human Development Index built by the arithmetic mean; H*: Human Development Index built by the IAEM function; P: inequality Penalisation Index; γ=1 is for decreasing penalisation; γ=0 is for constant penalisation; γ=-1 is for increasing penalisation.

Secondly, we calculate the indices from each dimension of the human development (income, education, health) by aggregating income quintiles. In this case we adopt the first step of the procedure of Foster et al. (2005) by putting $\varepsilon = 2$. (see Table 2). Table 2 shows some interesting results. The Income Index has the biggest percentage of cases with rank difference between the Index built by arithmetic mean (Z_Y) and the Index built by IAEM function (Z_Y*): the values of this pecentage are 62.50 per cent for decreasing penalisation, 18.75 per cent for consant penalisation and 43.75 per cent for increasing penalisation. In particular, for the case of decreasing penalisation Guinea and Nicaragua have the highest rank differences, respectively -6 and -4; for the constant penalisation, Guinea has the biggest rank difference (-4); finally for the case of increasing penalisation, countries with the highest rank differences are Guinea (-5) and Zambia (-4) (see Table A.2). The Education Index has the highest rank correlation between the Index built by arithmetic mean (Z_E) and the inequality penalisation index (P_E): the values are -0.69 for decreasing penalisation, -0.70 for constant penalisation and -0.73 for increasing penalisation. The most significant rank difference concerns Brazil (-6) in the case of decreasing penalisation. (see Table A.3). Further, about the Health Index, in the case of decreasing penalisation Brazil and Peru have the highest rank differences, respectively -4 and -5. In the other two cases of penalisation, the most significant rank difference concerns Peru with value -2 and Poland with value 2. (see Table A.4)

Table 2

	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$
Percentage of cases with rank difference (Z_Y-Z_Y*)	62.50	18.75	43.75
Rating Correlation between Z_Y and P_Y	-0.26	-0.62	-0.59
Spearman's Rank Correlation between Z_Y and P_Y	-0.46	-0.55	-0.60
Percentage of cases with rank difference (Z_E-Z_E*)	34.38	6.25	18.75
Rating Correlation between Z_E and P_E	-0.22	-0.43	-0.44
Spearman's Rank Correlation between Z_E and P_E	-0.69	-0.70	-0.73
Percentage of cases with rank difference (Z_H-Z_H*)	28.13	18.75	15.63
Rating Correlation between Z_H and P_H	0.07	-0.10	-0.02
Spearman's Rank Correlation between Z_H and P_H	-0.37	-0.15	-0.31

Z_Y: Income per capita Index built by the arithmetic mean; Z_Y*: Income per capita Index built by the IAEM function; Z_E: Education Index built by the arithmetic mean; Z_E*: Education Index built by the IAEM function; Z_H: Health Index built by the arithmetic mean; Z_H*: Health Index built by the IAEM function; P_Y: Inequality Penalisation Index for Income per capita; P_E: Inequality Penalisation Index for Education; P_H: Inequality Penalisation Index for Health; γ=1 is for decreasing penalisation; γ=0 is for constant penalisation; γ=-1 is for increasing penalisation.

Conclusions

Our goal has been to analyse the inequality aspects of Human Development Index and to propose a new aggregation function that adjusts it by considering inequality penalisation. We have taken into account inequality across dimensions and across groups and three laws of inequality penalisation: decreasing, constant and increasing. At the beginning, we have described the features of standard Human Development Index and after we have surveyed main analytical contributions regarding the inequality adjustment of Human Development Index; they are: the Human Development Report (1993), Anand and Sen (1995), Hicks (1997) , Foster et al. (2005) Stanton (2006), Grimm et al. (2008; 2010), Seth

(2009), the Human Development Report (2010). Successively, we have declined the basic properties of the Human Development Indices and also we have presented specific properties enjoyed by the aggregation function proposed: the Inequality Adjusted Exponential Mean (IAEM). This function is a specific case of the generalised mean.

Three are the innovative aspects of IAEM function not analysed by literature and not enjoyed by the Inequality Adjusted Human Development Index of Human Development Report 2010. Firstly, the domain of IAEM function is unlimited. Thanks to this property, it is possible to use all kinds of normalisation, as for example the standardisation where there are negative values; this fact enables the function to be flexible to the goal of the analysis. Secondly, IAEM function enjoys the property of *incomplete compensability*, that is for each dimension exists an upper bound of its decrease beyond which the same index value cannot be restored by increases in the other dimension; this is very realistic for the human development: for example Health Index's decreases can not be compensated infinitively by the increases of income index because there are the minimum level of health condition, behind which there is human underdevelopment. Thirdly, IAEM function fulfils the property of *level dependent inequality aversion*. According to this property with IAEM function it is possible to build three different rating and ranking classifications according to the laws of inequality penalisation, while other functions assume indirectly the decreasing penalisation. This property is relevant because in the development studies, there are different theories concerning the relationship between inequality and development. Thus, thanks to the introduction of a parameter that captures the sign of this relation, the researcher can choose the value of parameter more appropriate according to his theory.

Finally, we have applied the IAEM function to the database with 32 countries, developing and developed (Grimm et al. 2010). According to the results the Inequality Adjusted Human Development Index built by the IAEM function is significantly different from the standard Human Development Index built by the arithmetic mean, especially for the cases of decreasing and increasing penalisation. Moreover there is a negative correlation between the level of standard Human Development Index and the Inequality Penalisation Index, both in terms of rating and ranking.

Appendix

Table A.1

Country	Rating							Ranking									
	H	H*			P			H	H*			P			H - H*		
		$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$		$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$
Australia	0.952	0.9416	0.9493	0.9486	0.0104	0.0027	0.0034	1	2	1	1	26	28	27	-1	0	0
Canada	0.9478	0.9382	0.9451	0.9444	0.0096	0.0027	0.0034	2	3	2	13	27	27	28	-1	0	-11
Sweden	0.946	0.9440	0.9441	0.9438	0.0020	0.0019	0.0022	3	1	3	9	32	32	32	2	0	-6
Netherlands	0.9426	0.9354	0.9397	0.9391	0.0072	0.0029	0.0035	4	4	4	21	28	26	26	0	0	-17
Finnland	0.9402	0.9349	0.9375	0.9370	0.0053	0.0027	0.0032	5	5	5	6	30	29	30	0	0	-1
France	0.938	0.9309	0.9353	0.9347	0.0071	0.0027	0.0033	6	6	6	7	29	31	29	0	0	-1
Germany	0.929	0.9240	0.9263	0.9258	0.0050	0.0027	0.0032	7	7	7	24	31	30	31	0	0	-17
Italy	0.926	0.9118	0.9227	0.9218	0.0142	0.0033	0.0042	8	8	8	20	25	25	25	0	0	-12
USA	0.926	0.9055	0.9212	0.9194	0.0205	0.0048	0.0066	9	10	9	19	22	23	23	-1	0	-10
Spain	0.922	0.9076	0.9182	0.9172	0.0144	0.0038	0.0048	10	9	10	3	24	24	24	1	0	7
Poland	0.8648	0.8319	0.8570	0.8546	0.0329	0.0078	0.0102	11	11	11	23	19	22	22	0	0	-12
Brazil	0.838	0.7361	0.8141	0.8014	0.1019	0.0239	0.0366	12	12	12	12	6	10	11	0	0	0
Peru	0.7976	0.6669	0.7655	0.7433	0.1307	0.0321	0.0543	13	15	14	11	2	7	7	-2	-1	2
Colombia	0.7922	0.6945	0.7704	0.7593	0.0977	0.0218	0.0329	14	13	13	27	9	12	12	1	1	-13
Paraguay	0.7714	0.6664	0.7478	0.7336	0.1050	0.0236	0.0378	15	16	15	14	5	11	10	-1	0	1
Guatemala	0.744	0.6739	0.7250	0.7161	0.0701	0.0190	0.0279	16	14	16	28	14	14	17	2	0	-12
Indonesia	0.7378	0.6636	0.7183	0.7081	0.0742	0.0195	0.0297	17	17	17	8	13	13	14	0	0	9
Bolivia	0.734	0.6454	0.7085	0.6956	0.0886	0.0255	0.0384	18	18	18	18	10	9	9	0	0	0
Vietnam	0.7306	0.6295	0.7030	0.6879	0.1011	0.0276	0.0427	19	19	19	29	7	8	8	0	0	-10
Nicaragua	0.7194	0.5873	0.6850	0.6606	0.1321	0.0344	0.0588	20	22	20	15	1	6	3	-2	0	5
Kyrgyz Republic	0.7182	0.6008	0.6819	0.6630	0.1174	0.0363	0.0552	21	20	21	26	4	3	6	1	0	-5
South Africa	0.6698	0.5493	0.6264	0.6051	0.1205	0.0434	0.0647	22	23	23	10	3	1	2	-1	-1	12
India	0.645	0.5954	0.6287	0.6193	0.0496	0.0163	0.0257	23	21	22	17	16	17	19	2	1	6
Ghana	0.562	0.5102	0.5440	0.5311	0.0518	0.0180	0.0309	24	24	24	16	15	15	13	0	0	8
Madagascar	0.5584	0.4577	0.5209	0.4922	0.1007	0.0375	0.0662	25	26	25	31	8	2	1	-1	0	-6
Cameroon	0.517	0.4854	0.5038	0.4967	0.0316	0.0132	0.0203	26	25	26	2	20	20	20	1	0	24
Guinea	0.481	0.4057	0.4460	0.4239	0.0753	0.0350	0.0571	27	28	27	22	12	5	5	-1	0	5
Zambia	0.4662	0.3869	0.4304	0.4086	0.0793	0.0358	0.0576	28	29	29	32	11	4	4	-1	-1	-4
Cote d'Ivoire	0.4552	0.4349	0.4457	0.4400	0.0203	0.0095	0.0152	29	27	28	30	23	21	21	2	1	-1
Ethopia	0.3898	0.3592	0.3756	0.3641	0.0306	0.0142	0.0257	30	30	30	5	21	19	18	0	0	25
Mozambique	0.373	0.3397	0.3573	0.3445	0.0333	0.0157	0.0285	31	31	31	4	18	18	16	0	0	27
Burkina Faso	0.3728	0.3376	0.3558	0.3434	0.0352	0.0170	0.0294	32	32	32	25	17	16	15	0	0	7

H: Human Development Index built by the arithmetic mean; H*: Human Development Index built by the IAEM function; P: Inequality Penalisation Index; γ=1 is for decreasing penalisation; γ=0 is for constant penalisation; γ=-1 is for increasing penalisation.

Table A.2

	Rating							Ranking									
	Z_Y	Z_Y*			P_Y			Z_Y	Z_Y*			P_Y			Z_Y- Z_Y		
		$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$		$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$
Sweden	0.941	0.921	0.938	0.936	0.020	0.004	0.005	1	2	1	1	31	31	31	-1	0	0
Netherlands	0.941	0.917	0.937	0.935	0.024	0.004	0.006	2	3	2	2	29	30	30	-1	0	0
Finnland	0.937	0.922	0.934	0.933	0.015	0.003	0.004	3	1	3	3	32	32	32	2	0	0
Canada	0.935	0.906	0.930	0.928	0.030	0.005	0.007	4	4	4	4	27	27	27	0	0	0
Australia	0.934	0.904	0.929	0.927	0.030	0.005	0.007	5	6	5	5	26	26	26	-1	0	0
Germany	0.929	0.905	0.924	0.923	0.024	0.005	0.006	6	5	6	6	30	29	29	1	0	0
USA	0.927	0.888	0.920	0.918	0.039	0.007	0.009	7	8	7	7	20	25	25	-1	0	0
France	0.923	0.897	0.918	0.916	0.026	0.005	0.006	8	7	8	8	28	28	28	1	0	0
Italy	0.901	0.866	0.894	0.892	0.036	0.007	0.009	9	9	9	9	22	23	23	0	0	0
Spain	0.897	0.862	0.890	0.888	0.035	0.007	0.009	10	10	10	10	23	24	24	0	0	0
Poland	0.808	0.770	0.798	0.795	0.037	0.009	0.013	11	11	11	11	21	22	22	0	0	0
Brazil	0.769	0.662	0.739	0.726	0.107	0.030	0.043	12	12	12	12	7	9	11	0	0	0
Peru	0.730	0.602	0.690	0.671	0.128	0.040	0.059	13	15	13	13	2	3	3	-2	0	0
Guatemala	0.729	0.604	0.689	0.671	0.125	0.040	0.058	14	14	14	14	3	5	5	0	0	0
South Africa	0.729	0.605	0.689	0.670	0.124	0.040	0.059	15	13	15	15	4	4	4	2	0	0
Colombia	0.696	0.587	0.659	0.642	0.109	0.037	0.054	16	16	16	16	6	6	6	0	0	0
Paraguay	0.657	0.554	0.624	0.605	0.102	0.033	0.051	17	18	17	17	8	7	9	-1	0	0
Bolivia	0.624	0.550	0.598	0.585	0.073	0.026	0.039	18	19	18	19	10	12	14	-1	0	-1
Indonesia	0.612	0.560	0.594	0.585	0.052	0.018	0.027	19	17	19	18	15	16	17	2	0	1
India	0.604	0.522	0.573	0.558	0.082	0.031	0.047	20	20	20	20	9	8	10	0	0	0
Nicaragua	0.570	0.453	0.523	0.495	0.117	0.047	0.075	21	25	22	23	5	2	2	-4	-1	-2
Guinea	0.541	0.369	0.460	0.409	0.173	0.082	0.133	22	28	26	27	1	1	1	-6	-4	-5
Vietnam	0.535	0.504	0.523	0.517	0.031	0.012	0.019	23	21	21	21	25	21	21	2	2	2
Kyrgyz Republic	0.521	0.479	0.505	0.495	0.043	0.016	0.026	24	22	23	22	18	18	19	2	1	2
Cameroon	0.515	0.472	0.498	0.489	0.043	0.017	0.027	25	23	24	24	17	17	18	2	1	1
Cote d'Ivoire	0.511	0.471	0.495	0.486	0.040	0.016	0.025	26	24	25	25	19	20	20	2	1	1
Ghana	0.469	0.413	0.446	0.430	0.056	0.023	0.039	27	26	27	26	14	14	13	1	0	1
Zambia	0.454	0.395	0.428	0.411	0.059	0.026	0.043	28	27	28	32	12	13	12	1	0	-4
Burkina Faso	0.415	0.364	0.392	0.376	0.050	0.023	0.038	29	29	29	28	16	15	15	0	0	1
Madagascar	0.381	0.318	0.351	0.328	0.062	0.030	0.052	30	30	30	29	11	10	7	0	0	1
Mozambique	0.347	0.288	0.317	0.295	0.058	0.029	0.052	31	31	31	30	13	11	8	0	0	1
Ethopia	0.316	0.283	0.300	0.287	0.032	0.016	0.029	32	32	32	31	24	19	16	0	0	1

Z_Y: Income per capita Index built by the arithmetic mean; Z_Y*: Income per capita Index built by the IAEM function; P_Y: Inequality Penalisation Index for Income per capita; γ=1 is for decreasing penalisation; γ=0 is for constant penalisation; γ=-1 is for increasing penalisation.

Table A.3

	Rating							Ranking									
	Z_E	Z_E^*			P_E			Z_E	Z_E^*			P_E			Z_E- Z_E^*		
		$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$		$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$
Australia	0.988	0.994	0.988	0.988	-0.006	0.000	0.000	1	1	1	1	31	30	30	0	0	0
Canada	0.985	0.990	0.984	0.984	-0.005	0.000	0.000	2	2	2	2	30	29	29	0	0	0
Finnland	0.983	0.988	0.982	0.982	-0.005	0.000	0.000	3	3	3	3	27	27	27	0	0	0
Netherlands	0.979	0.985	0.979	0.979	-0.005	0.000	0.000	4	4	4	4	29	28	28	0	0	0
Sweden	0.969	0.975	0.969	0.969	-0.006	0.000	0.000	5	5	5	5	32	31	31	0	0	0
France	0.968	0.972	0.968	0.968	-0.004	0.000	0.000	6	6	6	6	26	26	26	0	0	0
USA	0.967	0.967	0.966	0.965	0.000	0.001	0.001	7	7	7	7	21	20	22	0	0	0
Italy	0.954	0.955	0.953	0.953	-0.001	0.001	0.001	8	8	8	8	24	23	23	0	0	0
Germany	0.954	0.955	0.953	0.953	-0.001	0.001	0.001	9	9	9	9	23	24	24	0	0	0
Poland	0.947	0.947	0.946	0.946	0.001	0.001	0.001	10	10	10	10	19	19	19	0	0	0
Spain	0.945	0.943	0.944	0.944	0.003	0.001	0.002	11	11	11	11	16	16	16	0	0	0
Kyrgyz Republic	0.925	0.929	0.925	0.925	-0.003	0.000	0.000	12	12	12	12	25	25	25	0	0	0
Brazil	0.891	0.816	0.877	0.872	0.076	0.014	0.020	13	19	14	14	1	2	2	-6	-1	-1
Peru	0.881	0.882	0.880	0.880	0.000	0.001	0.001	14	13	13	13	22	21	21	1	1	1
Bolivia	0.878	0.842	0.871	0.869	0.036	0.007	0.010	15	17	15	16	4	6	6	-2	0	-1
Colombia	0.873	0.864	0.870	0.870	0.008	0.002	0.003	16	14	16	15	12	14	14	2	0	1
Paraguay	0.859	0.857	0.858	0.858	0.002	0.001	0.002	17	15	17	17	17	17	17	2	0	0
South Africa	0.843	0.848	0.843	0.843	-0.005	0.000	0.000	18	16	18	18	28	32	32	2	0	0
Indonesia	0.838	0.824	0.834	0.833	0.014	0.004	0.005	19	18	19	19	8	12	13	1	0	0
Vietnam	0.814	0.812	0.813	0.812	0.002	0.001	0.002	20	20	20	20	18	18	18	0	0	0
Nicaragua	0.776	0.772	0.774	0.773	0.004	0.002	0.002	21	21	21	21	15	15	15	0	0	0
Guatemala	0.737	0.696	0.726	0.722	0.041	0.011	0.015	22	22	22	22	3	3	3	0	0	0
Madagascar	0.712	0.657	0.695	0.688	0.055	0.017	0.024	23	23	23	23	2	1	1	0	0	0
Zambia	0.671	0.650	0.664	0.661	0.021	0.006	0.009	24	24	24	24	6	7	7	0	0	0
India	0.654	0.643	0.651	0.649	0.011	0.004	0.005	25	25	25	25	10	11	12	0	0	0
Ghana	0.618	0.593	0.610	0.606	0.025	0.008	0.012	26	27	26	26	5	5	5	-1	0	0
Cameroon	0.613	0.600	0.608	0.606	0.013	0.005	0.007	27	26	27	27	9	8	9	1	0	0
Mozambique	0.472	0.471	0.471	0.471	0.000	0.001	0.001	28	28	28	28	20	22	20	0	0	0
Cote d'Ivoire	0.461	0.453	0.457	0.455	0.008	0.004	0.006	29	29	29	29	11	10	10	0	0	0
Ethopia	0.381	0.362	0.372	0.366	0.018	0.009	0.014	30	31	30	31	7	4	4	-1	0	-1
Guinea	0.375	0.368	0.371	0.369	0.007	0.003	0.006	31	30	31	30	14	13	11	1	0	1
Burkina Faso	0.250	0.243	0.246	0.243	0.007	0.004	0.007	32	32	32	32	13	9	8	0	0	0

Z_E: Education Index built by the arithmetic mean; Z_E^*: Education Index built by the IAEM function; P_E: Inequality Penalisation Index for Education; γ=1 is for decreasing penalisation; γ=0 is for constant penalisation; γ=-1 is for increasing penalisation.

Table A.4

	Rating							Ranking									
	Z_H	Z_H*			P_H			Z_H	Z_H*			P_H			Z_H - Z_H*		
		$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$		$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$	$\gamma = 1$	$\gamma = 0$	$\gamma = -1$
Australia	0.934	0.934	0.933	0.933	0.000	0.001	0.001	1	1	1	1	21	19	20	0	0	0
Sweden	0.928	0.928	0.927	0.927	0.000	0.001	0.001	2	2	2	2	23	23	25	0	0	0
Canada	0.924	0.924	0.923	0.923	0.000	0.001	0.001	3	3	3	3	24	24	24	0	0	0
France	0.923	0.923	0.922	0.922	0.000	0.001	0.001	4	4	4	4	25	20	21	0	0	0
Italy	0.923	0.923	0.922	0.922	0.000	0.001	0.001	5	5	5	5	26	21	22	0	0	0
Spain	0.923	0.923	0.922	0.922	0.000	0.001	0.001	6	6	6	6	27	22	23	0	0	0
Netherlands	0.907	0.907	0.906	0.906	0.000	0.001	0.001	7	7	7	7	28	27	29	0	0	0
Germany	0.904	0.904	0.903	0.903	0.000	0.001	0.001	8	8	8	8	29	25	28	0	0	0
Finnland	0.901	0.901	0.900	0.900	0.000	0.001	0.001	9	9	9	9	30	26	27	0	0	0
USA	0.884	0.872	0.881	0.880	0.012	0.003	0.004	10	10	10	10	12	15	16	0	0	0
Brazil	0.854	0.778	0.837	0.831	0.076	0.016	0.023	11	15	12	13	2	2	2	-4	-1	-2
Vietnam	0.842	0.817	0.837	0.835	0.025	0.005	0.007	12	12	13	12	4	9	10	0	-1	0
Poland	0.839	0.839	0.838	0.838	-0.001	0.001	0.001	13	11	11	11	31	28	30	2	2	2
Nicaragua	0.813	0.800	0.809	0.808	0.013	0.004	0.005	14	14	14	14	11	12	13	0	0	0
Colombia	0.806	0.807	0.806	0.805	-0.001	0.001	0.001	15	13	15	15	32	29	32	2	0	0
Paraguay	0.797	0.778	0.792	0.790	0.019	0.005	0.007	16	16	16	16	6	8	8	0	0	0
Peru	0.781	0.646	0.745	0.727	0.135	0.036	0.053	17	22	19	19	1	1	1	-5	-2	-2
Guatemala	0.765	0.759	0.762	0.762	0.005	0.002	0.003	18	17	17	17	16	17	17	1	1	1
Indonesia	0.763	0.744	0.757	0.755	0.018	0.005	0.007	19	18	18	18	8	10	9	1	1	1
Kyrgyz Republic	0.708	0.691	0.703	0.700	0.017	0.006	0.008	20	19	20	20	9	6	7	1	0	0
Bolivia	0.700	0.681	0.694	0.692	0.019	0.006	0.008	21	20	21	21	7	5	5	1	0	0
India	0.676	0.650	0.668	0.664	0.026	0.009	0.012	22	21	22	22	3	3	4	1	0	0
Ghana	0.598	0.584	0.593	0.590	0.015	0.006	0.008	23	23	23	23	10	7	6	0	0	0
Madagascar	0.583	0.572	0.579	0.577	0.012	0.004	0.006	24	24	24	24	13	11	11	0	0	0
Guinea	0.526	0.505	0.518	0.514	0.021	0.008	0.013	25	25	25	25	5	4	3	0	0	0
Ethopia	0.472	0.467	0.470	0.468	0.005	0.003	0.004	26	26	26	26	17	16	15	0	0	0
Burkina Faso	0.454	0.452	0.453	0.452	0.002	0.001	0.002	27	27	27	27	18	18	18	0	0	0
South Africa	0.437	0.431	0.434	0.432	0.007	0.003	0.005	28	28	28	28	15	14	14	0	0	0
Cameroon	0.423	0.423	0.422	0.422	0.000	0.001	0.001	29	29	29	29	22	32	31	0	0	0
Cote d'Ivoire	0.394	0.386	0.390	0.388	0.008	0.004	0.006	30	30	30	30	14	13	12	0	0	0
Mozambique	0.301	0.301	0.300	0.300	0.001	0.001	0.001	31	31	31	31	20	31	26	0	0	0
Zambia	0.274	0.274	0.273	0.273	0.001	0.001	0.001	32	32	32	32	19	30	19	0	0	0

Z_H: Health Index by the arithmetic mean; Z_H*: Health Index by the IAEM function; P_H: Inequality Penalisation Index for Health; γ=1 is for decreasing penalisation; γ=0 is for constant penalisation; γ=-1 is for increasing penalisation.

Table A.5

Country	Database			
Brazil	1996	Demographic and Health Survey (DHS)	1997	Living Standard Measurement Survey (LSMS)
Ethiopia	2000	Demographic and Health Survey (DHS)	2000	Welfare Monitoring/Income Consumption and Expenditure Survey
Guinea	1995	Demographic and Health Survey (DHS)	1999	Living Standard Measurement Survey (LSMS)
Ghana	1998	Demographic and Health Survey (DHS)	1998	Ghana Living Standard Survey No.4 Survey
Guatemala	1995	Demographic and Health Survey (DHS)	2000	Living Standard Measurement Survey (LSMS)
India	1999	Demographic and Health Survey (DHS)	1997	NSS Household Consumer Expenditure Survey (53rd Round)
Kyrgyz Republic	1997	Demographic and Health Survey (DHS)	1998	Living Standard Measurement Survey (LSMS)
Paraguay	1990	Demographic and Health Survey (DHS)	1998	Encueata Integrada De Hogares (Programa MECOVI)
Peru	2000	Demographic and Health Survey (DHS)	1994	Living Standard Measurement Survey (LSMS)
Burkina Faso	2003	Demographic and Health Survey (DHS)	2003	Enquete Prioritaire sur les Conditions de Vie des Menages (EP)
Bolivia	2003	Demographic and Health Survey (DHS)	2002	Living Standard Measurement Survey (LSMS)
Cote d'Ivoire	1999	Demographic and Health Survey (DHS)	1998	Enquete de Niveau de Vie des Menages (ENV)
Cameroon	2004	Demographic and Health Survey (DHS)	2001	Enquete Camerounaise auprues des Menages (ECAM)
Colombia	2005	Demographic and Health Survey (DHS)	2003	Encuesta de Calidad de Vida
Indonesia	2003	Demographic and Health Survey (DHS)	2000	Demographic and Health Survey (DHS)
Madagascar	1997	Demographic and Health Survey (DHS)	2001	Enquete auprues des Menages (EPM)
Mozambique	2003	Demographic and Health Survey (DHS)	2002	Inquerito Nacional aos Agregados Familiares sobre as Condiciones de Vida
Nicaragua	2001	Demographic and Health Survey (DHS)	2001	Encuesta Nacional de Hogares sobre Medicion de Nivel de Vida (EMNV)
South Africa	1998	Demographic and Health Survey (DHS)	2000	Income and Expenditure Survey
Vietnam	2002	Demographic and Health Survey (DHS)	2004	Living Standard Measurement Survey (LSMS)
Zambia	2002	Demographic and Health Survey (DHS)	2002	Living Standard Measurement Survey (LSMS)
Australia	2001	Luxembourg Income Study (LIS)		
Canada	2000	Luxembourg Income Study (LIS)		
Finnland	2000	Luxembourg Income Study (LIS)		
France	2000	Luxembourg Income Study (LIS)		
Germany	2000	Luxembourg Income Study (LIS)		
Italy	2000	Luxembourg Income Study (LIS)		
Netherlands	1999	Luxembourg Income Study (LIS)		
Poland	1999	Luxembourg Income Study (LIS)		
Spain	2000	Luxembourg Income Study (LIS)		
Sweden	2000	Luxembourg Income Study (LIS)		
USA	2000	Luxembourg Income Study (LIS)		

Source: Grimm M., et. al. (2010)

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