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A clockwork economy

A three-dimensional trade model
imbedded in the general equilibrium theory

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WORK IN PROGRESS, PLEASE DO NOT CITE

Abstract: *The analysis of South African trade data from 1993 until 2006 by simultaneously using a Grubel-Lloyd index, a measurement of marginal intra-industry trade and a revealed comparative advantage (RCA) indicator shows that South African imports and exports increased steadily and that the structural change of trade was characterised by specialisation. Whereas South Africa exports similar products to both China and the EU, it imports mainly high-tech goods from the EU and low-tech, labour-intensive goods from China. South African exports originate from a cluster of industries which are linked to the South African mining industry. These facts motivate the application of a multi-dimensional Heckscher-Ohlin-Samuelson trade model to explain the observed South African trade pattern. More specific a 3x3x3 trade model imbedded in the general equilibrium theory is constructed to predict trade flows between three stylised countries representing South Africa, the EU and China. Due to the dimensions of the model the concept of the endowment triangle is included in the model which makes a theoretical analysis simpler and visualises the results.*

Keywords: Heckscher-Ohlin trade model; general equilibrium theory; three-dimensional; South Africa; intra-industry trade; trade specialisation

JEL-Classification: F11; F14; F17

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Introduction

After the abolition of apartheid and the coming to power of the first democratic elected government in 1994, South Africa could participate more actively in the international community. One of the main macroeconomic policies of the new government was the liberalisation of the trade regime, which was, until then, very complex, anit-export-biased and protectionist. In 1995 South Africa became a founding member of the World Trade Organisation and is highly engaged in the Doha-Development Agenda. Moreover, a trade agreement with its major trading partner, i.e. the European Union (EU), was signed after years of intensive negotiations in 1999. At the same time the domestic economy got more exposed to international competition and especially the upsurge of China as an exporting country should be noted in this context. The effect of this trade policy on the trade performance of different industries is exciting indeed, because South Africa is a semi-industrialised country that inherited an industrial structure that was mainly shaped by decades of apartheid industrial policy, creating capital intensive industries and an abundance of less-skilled labour.

In general, a policy of reciprocal trade liberalisation will result into a higher degree of trade openness, due to increased imports and exports. On the industry level, however, it is not clear what is going to happen. One possibility is that industries with a comparative advantage will export more, whereas imports in other industries will increase and the production in these industries in turn decreases. If this is the case the trade structure will be characterised by specialisation, i.e. inter-industry trade exists. Theoretic models that describe and explain this phenomenon are the Ricardian and Heckscher-Ohlin-Samuelson trade models. The alternative is that in the same industries exports and imports increase simultaneously. This case is described by intra-industry trade. The theoretic explanation for this situation is based on monopolistic competition between firms, e.g. Krugman (1980) and Meltiz (2003). From this consideration it is crucial that the change in the South African trade pattern over time is deconstructed. Whereas the first part of this paper consists of a descriptive analysis, which motivates the following section, the second part applies a three-dimensional Heckscher-Ohlin-Samuelson trade model to explain the observed trade structure.

Although the evolution of South African trade pattern is already the subject of different papers, e.g. Parr (2000) and Isemonger (2000), the extension in the empirical part of this

paper is threefold. First of all a longer analysis period is used. In the empirical analysis a time period of fourteen years (1993-2006) is considered, a time period that could not have been covered by older papers. Further, the trade data are disaggregated for different trading partners. In this way it is possible to consider different trends in the trade pattern with the EU and China. The last extension is the deployed analysis method. Whereas earlier work used different measures, this paper develops a method that combines these measures. Through this approach a consistent analysis method comes about. The principal finding of the first section is the specialisation of the South African trade pattern with respect to the EU as well as China since its negotiated revolution.

In the second section the empirical results of the first section are explained by means of a neo-classical trade model. Due to the importance of the mining and quarrying industry for South Africa´s trade balance, an Heckscher-Ohlin-Samuelson model seems appropriate. Moreover, the observed difference between trade with the EU and China underpins the necessity to expand the two-dimensional textbook model into a three-factor, three-good, three-country model. With respect to a multi-dimensional Heckscher-Ohlin-Samuelson trade theory the academic discussion focussed for most on the generalisation of the price equalisation, the Heckscher-Ohlin, the Samuelson-Stolper and the Rybczynski theorem, e.g. Samuelson (1953), Chipman (1966 and 1969), Batra (1970), Jones and Scheinkman (1977), Chang (1979), Takayama (1981) and Ethier (1984). Due to the generality of these papers, they could only be applied to describe the basic structure of the model. One major problem in a multi-dimensional trade model is the notion of factor intensity, which was only discussed by Jones and Scheinkman (1977). This new notion of factor intensity will be picked up by this paper in the discussion of the model. However, the model, which is constructed this way, does not allow for the treatment of the factor endowment of and trade flows between countries. Therefore the theoretical part includes the concept of the endowment triangle and the triangles of diversification (Leamer 1987) to establish a model that explains the trade patterns between the considered countries based on their factor endowment.

South Africa´s trade structure

Trade data supplied by the South African Department for Trade and Industry (DTI 2007) form the basis of the trade analysis. The main advantage of this approach is that these data are

available online and the results of this paper can thus be reconstructed without any difficulties. In accordance with other researchers, e.g. Isemonger (2000) and Parr (2000), the 4-digit level of the Harmonised System Classification is used and each class is regarded as an industry. Furthermore, use was made of GDP figures from Statistics South Africa (2007) to calculate annual import and export shares. As a result a period between 1993, the year before the first general democratic elections, and 2006 is covered by the data. Concerning trade partners, regional aggregates are used. The European Union aggregate includes all 27 countries, the member states in 2006, for the whole period and besides China also Hong Kong, Macao and Taiwan are part of the China aggregate. Since every sample contains at least thousand industries, the following sectors are used.

Description	HS-code
Animal and animal products	0100-0599
Vegetable products	0600-1599
Foodstuffs	1600-2499
Mineral products	2500-2799
Chemicals and allied industries	2800-3899
Plastics and rubbers	3900-4099
Raw hides, skins, leather and furs	4100-4399
Wood and wood products	4400-4999
Textiles	5000-6399
Footwear and headgear	6400-6799
Stone and glass	6800-7199
Metals	7200-8399
Machinery and electrical	8400-8599
Transportation[2]	8600-8999
Miscellaneous	9000-9799
Service[3]	9800-9999

Table 1: Sector aggregates
Source: Foreign Trade On-Line: 2007

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[2] The name of this aggregate industry is rather misleading, because it contains only industries of the transportation equipment industry and not of the transport industry.
[3] This industry aggregate contains only the classes 9801: Original equipment components for motorvehicles, 9991: Postal packages, not classified and 9999: Household effects & code 999999 (mostly platinum) and cannot be interpreted as services.

To analyse the changed trade structure this paper develops a new method as visualised in Figure 1. This method is based upon earlier work of A. G. Isemonger (2000) and R. G. Parr (2000) combined with a Grubel-Lloyd index of intra-industry trade (Grubel and Lloyd 1971). The Grubel-Lloyd index (GL) of each industry i can be calculated for a reference year (1993) and is defined as:

$$GL_i = \frac{(X_i + M_i) - |X_i - M_i|}{X_i + M_i} = 1 - \frac{|X_i - M_i|}{X_i + M_i} \, .$$

The absolute difference between the imports and exports of industry i indicates the level of inter-industry trade, i.e. the exports of industry i that are not matched by the imports of this industry and vice versa. This index is normalised by means of the value of overall trade and the value of this index ranges between zero and one. The used cut-off value to distinguish between industries with an intra-industry trade pattern and industries with an inter-industry trade structure is 0.65. Industries with an index value above this cut-off value are allocated to the intra-industry trade group. All other industries are regarded as industries with an inter-industry trade pattern, i.e. due to the used cut-off value, industries which import at least twice as much as they export or vice versa.
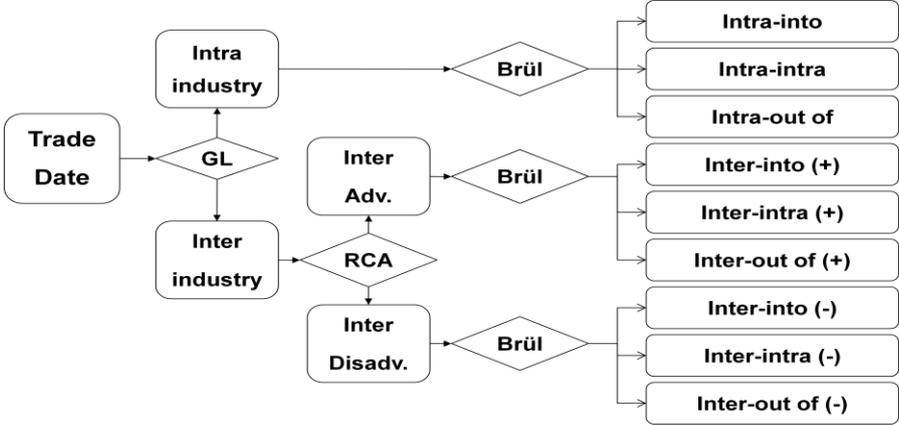
Figure 1: Analytic classification of industries

The next step is to distinguish in the inter-industry group between industries with a comparative advantage, i.e. a trade surplus, and those with a comparative disadvantage. By means of a Revealed Comparative Advantage index (RCA) it is possible to differentiate between these two groups. In order to keep the data volume as simple as possible, an RCA-

index is used that does not require the collection of extra data. For this analysis a RCA-index developed by Siebert (2000) was used. This index is defined as:

$$RCA_i = \left[\frac{X_i - M_i}{X_i + M_i} - \frac{\sum(X_i - M_i)}{\sum(X_i + M_i)} \right] \cdot \frac{100}{1 - \frac{\sum(X_i - M_i)}{\sum(X_i + M_i)}} .$$

Corresponding to this notation, a competitive industry is an industry with a higher relative level of net trade than the overall economy, i.e. the RCA-index is positive. Thus for the reference year this method results in the construction of three different classes: industries with a intra-industry trade pattern, industries with an inter-industry trade pattern and a comparative advantage, and industries with an inter-industry trade pattern and a comparative disadvantage.

To analyse the structural evolution of these classes a measure developed by Brülhart (1994) is applied. The basis of this measure is the increase or decrease of trade flows between the reference year and the end year, in this case 2006. The indicator is defined as:

$$D_i = \frac{\Delta X_i - \Delta M_i}{|\Delta X_i| + |\Delta M_i|} .$$

The Brülhart measure constructs three classes, namely specialisation into, intra-industry change and specialisation out of. All industries with export growing substantially faster than imports are allocated to the specialisation into class, whereas in the opposite case, they are listed in the specialisation out of class. Once again, the cut-off value is 0.65 and -0.65, respectively. The intra-industry change class contains all industries for which imports and exports changed similarly over time. Using this technique all industries can be allocated to one of the nine classes.

Before going into detail with respect to trade relations with the EU and China the overall trade characteristics of South Africa are discussed. In 1993 the overall Grubel-Lloyd index was 26.73, i.e. roughly 27% of total trade could be defined as intra-industry trade. On the disaggregated industry level around 17% of all industries had intra-industry trade patterns, whereas less than one third of the remaining 83% had a positive trade balance. In 1993 the main exports originated within the "stone and glass", the "mineral products" and the "metals" sectors. Simultaneously many products were imported from the "machinery and electrical", the "plastics and rubbers", the "transportation" and the "chemicals and allied industries" sectors. The overall trade balance was positive with Rand 14.05 billion. This was the result

of exports in value of Rand 37.89 billion combined with imports that amounted to Rand 59.84 billion.

Figure 2 shows the evolution of the export and import shares starting in 1993 until 2006. This line graph indicates the opening up of the South African economy. Whereas in 1993 the export and import share were both below 10%, they steadily increased, except for the year 2003, and reached respectively a value of 33.85% and 39.70% in 2006.
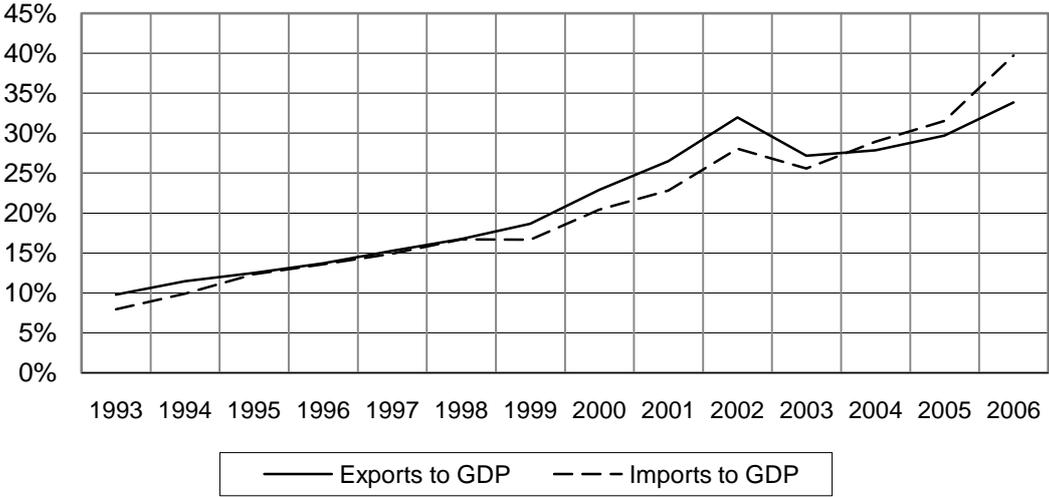
Figure 2: South African export and import shares over time
Source: Own calculations based on DTI-data (2007) and Statistics South Africa (2007)

This opening up was mainly due to the reintegration of South Africa into the global economy. After the coming to power of the African National Congress (ANC) other countries suspended their sanctions against South Africa. At the same time the new government actively negotiated trade agreements, e.g. the South African membership in the World Trade Organisation and the bilateral trade agreement with the EU, the Trade, Development and Cooperation Agreement.

In 2006 the export value amounted to Rand 396.48 billion, whereas products in worth of 465.04 were imported. The Grubel-Lloyd index increased to 30.59, but the trade structure did not change much. The main exports were still coal, briquettes, non-crude oil, diamonds, gold, platinum and ferro-alloys. Imports were still composed of manufactures. During this period, however, the import of fuel (e.g. crude oil and coal) increased and was responsible for the trade deficit of the "mineral products" aggregate.

What can be said about the trade pattern with the EU and about that with China? With respect to trade relations with the EU South Africa was a net importer in 1993. Moreover, the EU was South Africa´s biggest trading partner. Around 23% of all exports went into, whereas 42% of all imports originated from the EU. The "vegetable products" (9.40% of all exports to the EU), the "mineral products" (20.64%), the "stone and glass" (13.48%) and the "metals" (11.58%) sectors were the aggregates with the highest export shares, whereas the import shares of the "chemicals and allied industries" (16.13%), the "machinery and electrical" (34.65%) and the "transportation" (12.66%) sectors were the highest. This trade pattern resulted in an overall Grubel-Lloyd index of 19.84, i.e. around one fifth of total trade with the EU was intra-industry trade.

If one applies the analysis method described above specialisation is observable. Of all industries 43% had a trade deficit in 1993 and within the observed period specialisation out of occurred. Most of these industries could be found within the "chemicals and allied industries", the "metals", the "machinery and electrical" and the "miscellaneous" sectors. Simultaneously, around 7% of all industries displayed inter-industry trade with a trade surplus that increased during this period. This trend was observed mainly within the "vegetable products", the "mineral products" and the "metals" aggregates. Concerning the "metals" group, specialisation into was linked with raw materials, whereas specialisation out of could be observed for products of more complexity. Due to these observed changes it is not surprising that the Grubel-Lloyd index changed little and had a value of 19.71 in 2006. In the same year the EU absorbed 31.87% of all South African exports and supplied 34.65% of its imports.

With regard to China one can observe the increasing importance of this region during the sample period. Whereas China was still a net importer of South African products in 1993, since 1994 the trade balance was in permanent deficit. In 1993 6.06% of South African exports went to China and China supplied 7.02% of South Africa´s imports. Over time the export share remained nearly constant, but South Africa obtained 12.23% of its imports from China in 2006. The Grubel-Lloyd index of the year 1993 was 4.62, a very low figure indeed, meaning that only a fraction (4.62%) of trade with China was characterised by an intra-industry pattern. In this year the main exports mainly comprised commodities of the "mineral

products" (20.05%) and the "metals" (41.07%) aggregates, whereas imports constituted of "textiles" (20.75%) and "machinery and electrical" (38.03%) products. During the period 1992-2006 79.40% of all industries specialised out of, whereas specialisation into took place in around 13% of all industries. An increase of comparative disadvantageous industries happened in all aggregates. The main specialisation into occurred within the "mineral products", the "chemicals and allied industries" and the "metals" sectors. Therefore it is not surprising that the Grubel-Lloyd index even decreased and was 3.27 in 2006.

In conclusion South Africa´s economy opened up for global markets and participated in them and a clear specialisation took place. Its trade structure got more specialised. On the one hand, South Africa remained mainly a supplier of mining and quarrying products as well as of products of the basic iron and steel industries. On the other hand, imports originated in the "machinery and electrical" sector and other manufactures such as chemicals (EU) and textiles (China). Within the "machinery and electrical" import from Europe and China are partly overlapping. Both country groups supplied South Africa with office machines, and telecommunication consumer goods, e.g. telephone sets.

At this point it will show to be interesting to also consider the production structure of these exports and imports. If these products use different production factors more intensively, this is a clear indication for comparative advantages described by the Heckscher-Ohlin trade model. Therefore, the classification of Edwards (2001), which compares capital-labour ratios, is included. According to this classification both electrical and non-electrical machinery, as well as textiles are allocated to the labour-intensive sector. Chemicals are considered capital-intensive products, whereas mining and quarrying are regarded as primary products for which no information concerning factor intensity is given. However, iron and steel, and non-ferrous metal basic industries are assigned to be capital intensive industries. Furthermore, it is possible to distinguish between low- and high-technology manufactures. By means of the classification used by Lall (2000), which marks textiles and metals as low-technology manufactures and identifies office and telecommunication equipment as high-technology products, a remarkable difference between South African exports and imports can be established. One can claim that the EU supplied predominantly high-technology manufactures, whose production labour intensive and capital-intensive products, whereas

China focussed more on low-technology consumer goods[4]. South Africa exports primary products of the mining and quarrying industries and low-technology capital-intensive manufactures. Based on these findings a theoretical model thus has to keep account with these different production methods of imports and exports. Such a theoretical model that could explain the observed trade model is the topic of the following sections.

The basic structure of the model

The results of the former section show unambiguously that, in order to model the South African trade pattern, a theoretical framework that explains inter-industry trade has to be used. Taking a closer look at the pure theory of international trade this means that either a Ricardian trade model or an Heckscher-Ohlin trade model would be appropriate. Whereas in the first model comparative advantages of industries exist due to differences in the production technology, in the latter model these advantages are caused by different factor endowments. This paper opts for an Heckscher-Ohlin approach based on three arguments. First of all the mining and quarrying industries are South Africa´s major exporters. The rich soil and the endowment with natural resources shaped doubtlessly South Africa´s trade pattern. Secondly it seems rather awkward in the face of a globalised world and the historical link between South Africa and the EU to assume different production techniques. Finally this paper also wants to incorporate the use of different production factors, e.g. skilled and less-skilled labour.

Regarding these production factors it seems necessary to consider three different production factors. Besides the distinction between less-skilled (L) and skilled labour (H), capital linked to the cluster of industries within the mining and quarrying industries (K) is also considered. This approach is justified by the claim that these industries became more and more capital intensive, because of the increasing depths where minerals are extracted.

Although there is some overlapping between the South Africa-EU and the South Africa-China trade pattern, these countries cannot be treated as one economic region, i.e. the rest-of-the-world. Hence the model will establish trade relations between three economies, South Africa

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[4]   With respect to the high-technology manufactures imports originating from China, these products are probably assembled in China, but the high-technology parts of these products originate from more industrialised countries.

as an exporter of mining and quarrying products, the EU as a supplier of skilled labour intensive goods and China as an exporter of less-skilled labour intensive products.

Based upon the paper by Paul A. Samuelson (1953) the two dimensional Heckscher-Ohlin trade model can be expanded to a three-dimensional trade model. Additional assumptions in line with other mathematical trade models have to be made. First of all, the three countries produce a positive amount of three different goods in both the autarkic and the free-trade situation. Moreover, the production of these three goods requires the positive input of three production factors and each country uses the same production technique. Independent of its factor endowment the production within a country can be written as:

$$X_i = X^i(K_i, L_i, H_i) \text{ and } i \in \{1,2,3\} \text{ or its unit isoquant}$$

$$I = X^i(k_i, l_i, h_i) \text{ with } k_i = \frac{K_i}{X_i}, l_i = \frac{L_i}{X_i} \text{ and } h_i = \frac{H_i}{X_i}.$$

For the mathematical model the following notation is used: K_i is the amount of capital, L_i is the amount of less-skilled labour and H_i is the amount of skilled labour used in industry i. Each industry i has a certain positive amount of output X_i and the coefficients of production of the same industry are k_i, l_i and h_i. The production function of industry i is notated as X^i. The price of a good produced by industry i is given by p_i and w_K, w_L and w_H are the factor prices.

The production techniques are characterised by constant returns to scale, i.e. homogeneous production functions of the first order. These production functions are smooth, i.e. the assumption of diminishing marginal returns for each factor.

$$\frac{\partial X_i}{\partial K_i} > 0 \text{ and } \frac{\partial X_i^2}{\partial^2 K_i} < 0; \frac{\partial X_i}{\partial L_i} > 0 \text{ and } \frac{\partial X_i^2}{\partial^2 L_i} < 0 \text{ and } \frac{\partial X_i}{\partial H_i} > 0 \text{ and } \frac{\partial X_i^2}{\partial^2 H_i} < 0.$$

On the goods as well as the factor markets perfect competition is assumed. With respect to the consumers the assumption is made that they have the same utility function, in particular homothetic tastes.

$$w_K = p_i \frac{\partial X^i(k_i, l_i, h_i)}{\partial K_i}, w_L = p_i \frac{\partial X^i(k_i, l_i, h_i)}{\partial L_i} \text{ and } w_H = p_i \frac{\partial X^i(k_i, l_i, h_i)}{\partial H_i}.$$

Within a country factors can move freely between industries and will favour the industry with the highest wage. Due to the assumption of perfect competition, the marginal productivity of each factor will equalise between the industries.

Another consequence of this assumption is that none of the industries make any pure profit, they just break even. This zero-profit condition is described by following equation:

$$p_i = k_i w_K + l_i w_L + h_i w_H.$$

With respect to the consumption side, the demand for goods is given as a function of factor and goods prices.

$$X_i = D^i(p_1, p_2, p_3, w_K, w_L, w_H)$$

If one assumes inelastic factor supply, the factor endowment is constant and can be written as:

$$K = \overline{K}, L = \overline{L}, H = \overline{H}\;.$$

In the autarkic situation production equals consumption and the national income is spent on consumption or:

$$p_1 X_1 + p_2 X_2 + p_3 X_3 = w_K \overline{K} + w_L \overline{L} + w_H \overline{H}.$$

It was shown by Takayama (1981) that this 3x3-dimensional model is solvable and has a unique solution if the determinant of the coefficient matrix is bigger than zero. This condition is assumed to be fulfilled, because each industry uses a different technology, i.e. input coefficients between industries are not interrelated. Furthermore, the arrangement of the industries is arbitrary. Therefore it has to hold that:

$$\begin{vmatrix} k_1 & l_1 & h_1 \\ k_2 & l_2 & h_2 \\ k_3 & l_3 & h_3 \end{vmatrix} > 0.$$

Thus, using the general equilibrium theory, it is possible to describe the basics of the developed model. The following section addresses the problem of factor intensity within this model.

Factor intensity: a new approach

One major problem of describing possible outcomes for the above model is the notion of factor intensity. Within a textbook 2x2x2-model "[…] *a double bilateral comparison is involved, i.e. a ratio of two factors compared between two industries.*" (Jones and Scheinkman 1977: 912) From the moment on that three or more factors are involved in the production of one good this notion of factor intensity is confusing. As pointed out by Jones and Scheinkman (1977) two equivalent alternatives can be used. One possibility is the use of the distributive share of a factor, θ_{ij}, i.e. the share of the product price p_i that is used to remunerate the factor j. The distributive share of a factor is defined as:

$$\theta_{iK} = \frac{k_i w_K}{p_i}, \theta_{iL} = \frac{l_i w_L}{p_i}, \theta_{iH} = \frac{h_i w_H}{p_i}, 0 < \theta_{ij} < 1 \text{ and } \sum_j \theta_{ij} = 1 \,.$$

Another approach is to define factor intensity by means of a comparison of the fraction of the total supply of a factor employed in a specific industry, λ_{ij}, or by definition:

$$\lambda_{iK} = \frac{k_i X_i}{\overline{K}} = \frac{K_i}{\overline{K}}, \lambda_{iL} = \frac{L_i}{\overline{L}}, \lambda_{iH} = \frac{H_i}{\overline{H}} \text{ and } \sum_i \lambda_{ij} = 1.$$

If the work of Batra and Casas (1976) is included, the distributive share of a factor can be reinterpreted into the elasticity of the good´s price with respect to the change of that factor´s price. In their paper the authors showed that:

$$\sum_j \theta_{ij} \dot{w}_j = \dot{p}_i \text{ with } \dot{w}_j = \frac{dw_j}{w_j} \text{ and } \dot{p}_i = \frac{dp_i}{p_i} \,.$$

This equation can now be used to consider the change of only one factor price, e.g. capital. The equation can thus be rewritten into:

$$\theta_{iK} \frac{dw_K}{w_K} = \frac{dp_i}{p_i} \Rightarrow k_i \frac{w_K}{p_i} \frac{dw_j}{w_K} = \frac{dp_i}{p_i} \Rightarrow k_i = \frac{dp_i}{dw_K} \,.$$

If the equation for k_i is now substituted in the equation of the share of the product price that is used to remunerate capital, following equation is obtained:

$$\theta_{iK} = \frac{k_i w_K}{p_i} = \frac{dp_i w_K}{dw_K p_i} = \frac{\frac{dp_i}{p_i}}{\frac{dw_K}{w_K}} \text{ with } \sum_j \theta_{ij} = 1 \text{ and } 0 < \theta_{ij} < 1.$$

So the distributive share of a factor can be reinterpreted as the elasticity of the goods price with respect to the change of the factor price.

After this preparatory work the idea of factor intensity can now be completed. For this purpose the distributive share of a factor j in the economy as whole, i.e. the share of the national income that can be attributed to factor j, is needed. The definition of this share is:

$$\alpha^K = \frac{w_K \bar{K}}{w_K \bar{K} + w_L \bar{L} + w_H \bar{H}} \, ,$$

$$\alpha^L = \frac{w_L \bar{L}}{w_K \bar{K} + w_L \bar{L} + w_H \bar{H}} \, ,$$

$$\alpha^H = \frac{w_H \bar{H}}{w_K \bar{K} + w_L \bar{L} + w_H \bar{H}} \, ,$$

$$\sum_j \alpha^j = 1 \, .$$

It is also possible to define the share of the production value of industry i in total production as:

$$\alpha_i = \frac{p_i X_i}{\sum_i p_i X_i} \text{ with } \sum_i \alpha_i = 1 \, .$$

If the ratio of the distributive share of a factor j within a specific industry i is bigger than its overall distributive share, this industry is said to use this factor intensively. This ratio equals the ratio of the fraction of the total supply of a factor employed in a specific industry with the share of the production value of that specific industry. The ratio is given by following equation:

$$z_{ij} = \frac{\theta_{ij}}{\alpha^j} = \frac{\lambda_{ij}}{\alpha_i} \, ,$$

$$\text{e.g. } z_{1K} = \frac{\theta_{1K}}{\alpha^K} = \frac{\frac{k_1 w_K}{p_1}}{\frac{w_K \bar{K}}{w_K \bar{K} + w_L \bar{L} + w_H \bar{H}}} = \frac{\frac{K_1}{X_1} \cdot \frac{1}{p_1}}{\frac{\bar{K}}{w_K \bar{K} + w_L \bar{L} + w_H \bar{H}}} = \frac{\frac{K_1}{\bar{K}}}{\frac{p_1 X_1}{p_1 X_1 + p_2 X_2 + p_3 X_3}} = \frac{\lambda_{1K}}{\alpha_1} \, .$$

The index of intensities, z_{ij}, can be ranked for each of the three industries. In those industries where it is bigger than unity, it indicates that that specific industry uses the respective factor intensively. Furthermore, it is clear that at least one intensity index per industry has to be bigger than unity, because both $\sum_j \alpha^j = 1$ and $\sum_j \theta_{ij} = 1$. Under the assumption that each industry only uses one production factor intensively, the following arbitrary ranking can be established:

$$z_{1K} > 1 > z_{1L} > z_{1H} \text{ ,}$$

$$z_{2L} > 1 > z_{2H} > z_{2K} \text{ ,}$$

$$z_{3H} > 1 > z_{3K} > z_{3L} \text{ .}$$

Based on this ranking, the following disequilibria have to hold:

$$z_{1K} > 1 > z_{2K}, z_{3K} \text{ ,}$$

$$z_{2L} > 1 > z_{1L}, z_{3L} \text{ ,}$$

$$z_{3H} > 1 > z_{1H}, z_{2H} \text{ .}$$

The mathematical solution of the above described model is, however, ambiguous. Therefore, the next section takes a closer look at the endowment of the different countries and offers a graphical solution for the model.

Country positions within the endowment triangle

Thus far this paper described an abstract model without making any assumptions concerning the involved economies. This section corrects this shortcoming by means of a graphical solution. Although the model could be graphically represented in a three-dimensional space, it is possible to simplify it into a two-dimensional problem. This is done by using the concept of the endowment triangle. The remaining of this paper will construct such an endowment triangle and integrate endowment and production vectors. The result of this exercise is a comprehensive, graphical model that explains trade flows between different countries.

To define the factor endowment of the countries, this paper uses the concept of the endowment triangle, as described by Leamer (1987). The endowment triangle displays the relative endowments with respect to three factors within a 2-dimensional space. Figure 3 helps to understand the concept of the endowment triangle and the transformation from a three-dimensional to a two-dimensional space. A three-dimensional factor space is based upon three orthogonal axes, each representing a specific factor. Through this representation each factor endowment is represented by a three-dimensional vector. If, however, a plane is constructed in the three-dimensional factor space that intersects all three axes, i.e. a plane in the positive orthant, two-dimensional vectors can be used as endowment vectors. The transformation is conducted in the following way. First of all, a plane is constructed, in

Figure 3 this plane is defined by the points (1,0,0), (0,1,0) and (0,0,1). This plane is our endowment triangle. Furthermore, one of these points will become the new origin, in this case the point (1,0,0) will function as origin. Combining the new origin with the remaining points, two vectors can be constructed, v_1 and v_2, which results in this case in:

$$\vec{v_1} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

$$\vec{v_2} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

The three vertices of the endowment triangle can now be rewritten into:

$$(1,0,0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot \vec{v_1} + 0 \cdot \vec{v_2} = (0,0),$$

$$(0,1,0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 1 \cdot \vec{v_1} + 0 \cdot \vec{v_2} = (1,0),$$

$$(0,0,1) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot \vec{v_1} + 1 \cdot \vec{v_2} = (0,1).$$
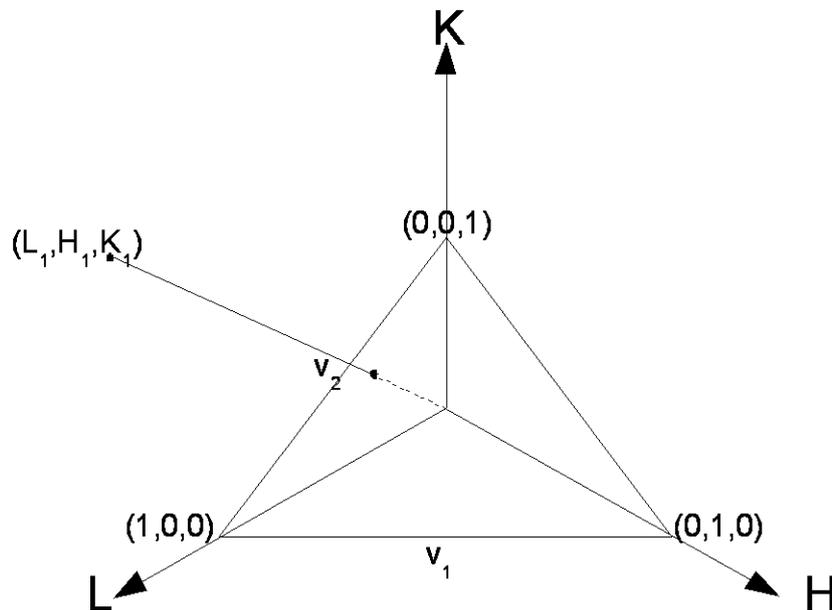
Figure 3: The construction of an endowment triangle

Every line defined by the original origin of the three-dimensional factor space and a three-dimensional endowment point, e.g. (L_1, H_1, K_1), will intersect the endowment triangle in a

specific point. Moreover, each point on this line will have the same ratios of factors. The point of intersection will now be used to represent the original three-dimensional factor endowment and is defined as:

$$\begin{pmatrix} L_1 \\ H_1 \\ K_1 \end{pmatrix} X = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha \cdot \overrightarrow{v_1} + \beta \cdot \overrightarrow{v_2} = (\alpha, \beta),$$

$$\begin{cases} L_1 X = 1 - \alpha - \beta \\ \quad H_1 X = \alpha \\ \quad K_1 X = \beta \end{cases}.$$

Due to the introduced vector notation, it is thus possible to simplify the three dimensional space to a two dimensional one. Each corner of the endowment triangle represents one production factor. A main feature of this representation is "[…] that *every endowment point on a straight line emanating from one corner of the triangle has the same ratio of the other two factors.*" (Leamer 1987: 964, original italics) This will be shown by means of the representation of point (L_1, H_1, K_1) in the endowment triangle of Figure 4.

Point C, (α, β), in Figure 4 is the two dimensional representation of the three dimensional point (L_1, H_1, K_1). The ratio of L and K in this point is given by the distance BC, which equals β, relative to the distance BD, which equals 1-α as long as the endowment triangle is an equilateral triangle, and can be obtained by following calculation:

$$\frac{\beta}{1-\alpha} = \frac{K_1 X}{1 - H_1 X}, \text{ with } 1 = X(L_1 + H_1 + K_1),$$

$$\frac{\beta}{1-\alpha} = \frac{K_1}{L_1 + K_1}.$$

It is easy to see that every point on the AH-line has the same ratio of L and K as point C. Using the same technique, it is straightforward that the other ratios for point C are:

$$\frac{\alpha}{1-\beta} = \frac{H_1}{L_1 + H_1},$$

$$\frac{\beta}{\alpha+\beta} = \frac{K_1}{H_1 + K_1}.$$
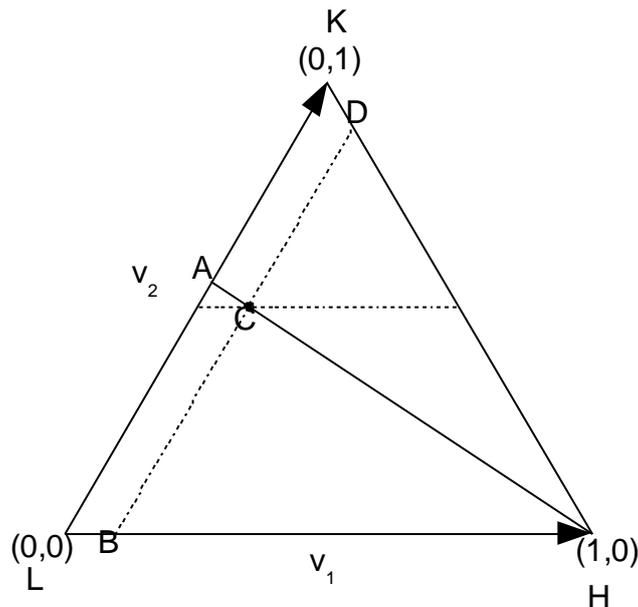
Figure 4: Endowment triangle

This concept is now applied on a theoretical world of three countries and three production factors. By means of scaling, it would be possible to position the total endowment of the three countries, i.e. the world, in the middle of the endowment triangle. The endowment of the world with capital, less-skilled labour and skilled labour can now be plotted in the endowment triangle. The relative endowments of the world, i.e. the K/(H+K), L/(K+L) and H/(K+H) ratios, are represented in Figure 5 by the points A, B and C, respectively. Furthermore, the dashed lines emanating from the three vertices divide the endowment triangle into six regions with different basic features. Because the interpretation for each region is straight forward, only the endowment relationship between region 1 and the representative world will be discussed. The interpretation of the other regions is left to the reader. All countries with a factor endowment point within region 1 have a higher endowment ratio for K/(H+K) than the world, because each possible line that emanates from the L-corner and that goes through region 1 cuts the K-H-line in a point closer to K than point A. The same is true with respect to the H-corner, the K-L-line and point B, i.e. all countries in region 1 have a higher K/(L+K) ratio than the world. It is also easily noticed that all lines that connect points within region 1 with the K-corner cut the L-H-line somewhere between point C and H, which implies a higher H/(L+H) ratio than the world for region 1.
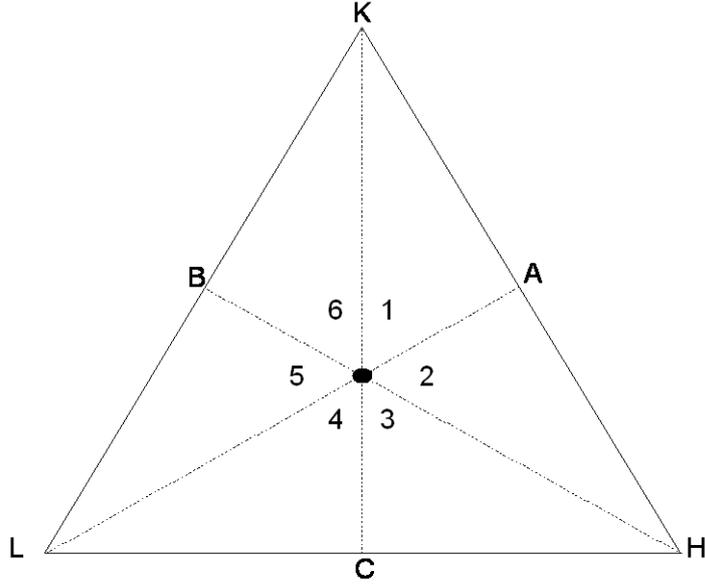
Figure 5: Regions in the endowment triangle

Due to the fact that the emphasis of this paper lies on explaining trade between South Africa (ZAF), the European Union (EU) and China (CH), the basic assumptions concerning the factor endowment of these countries should be made explicit. As already mentioned above, the three production factors considered are less-skilled labour (L), skilled labour (H) and capital bound to the Minerals-Energy Complex (K). Moreover, the world (W) consists only of the three economies already mentioned. Specific assumptions about factor endowment are given by following disequilibria:

$$\left(\frac{K}{H}\right)^{ZAF} > \left(\frac{K}{H}\right)^{CH} > \left(\frac{K}{H}\right)^{W} > \left(\frac{K}{H}\right)^{EU},$$

$$\left(\frac{K}{L}\right)^{ZAF} > \left(\frac{K}{L}\right)^{W} > \left(\frac{K}{L}\right)^{EU} > \left(\frac{K}{L}\right)^{CH},$$

$$\left(\frac{H}{L}\right)^{EU} > \left(\frac{H}{L}\right)^{ZAF} > \left(\frac{H}{L}\right)^{W} > \left(\frac{H}{L}\right)^{CH}.$$

These disequilibria can be rewritten into:

$$\left(\frac{K}{H+K}\right)^{ZAF} > \left(\frac{K}{H+K}\right)^{CH} > \left(\frac{K}{H+K}\right)^{W} > \left(\frac{K}{H+K}\right)^{EU},$$

$$\left(\frac{K}{L+K}\right)^{ZAF} > \left(\frac{K}{L+K}\right)^{W} > \left(\frac{K}{L+K}\right)^{EU} > \left(\frac{K}{L+K}\right)^{CH},$$

$$\left(\frac{H}{L+H}\right)^{EU} > \left(\frac{H}{L+H}\right)^{ZAF} > \left(\frac{H}{L+H}\right)^{W} > \left(\frac{H}{L+H}\right)^{CH}.$$

This rewriting is based upon the following property:

$$\frac{a}{b} > \frac{c}{d}$$

$$\Rightarrow ad > bc$$

$$\Rightarrow ad + ac > bc + ac$$

$$\Rightarrow \frac{c+d}{c} > \frac{b+a}{a}$$

$$\Rightarrow \frac{a}{b+a} > \frac{c}{c+d}.$$

A possible endowment situation of the three countries is displayed in Figure 6. In this figure South Africa is the country with the highest capital-skilled labour ratio, which is higher as the world endowment. Also the capital-skilled labour ratio of China is higher than the world ratio and only the EU has a lower ratio, due to its rich endowment with skilled labour. China is the country with the highest less-skilled labour ratio, both with respect to capital and skilled labour. Both the EU and South Africa have a less-skilled labour-skilled labour ratio higher than the world ratio, with the highest ratio of the EU.
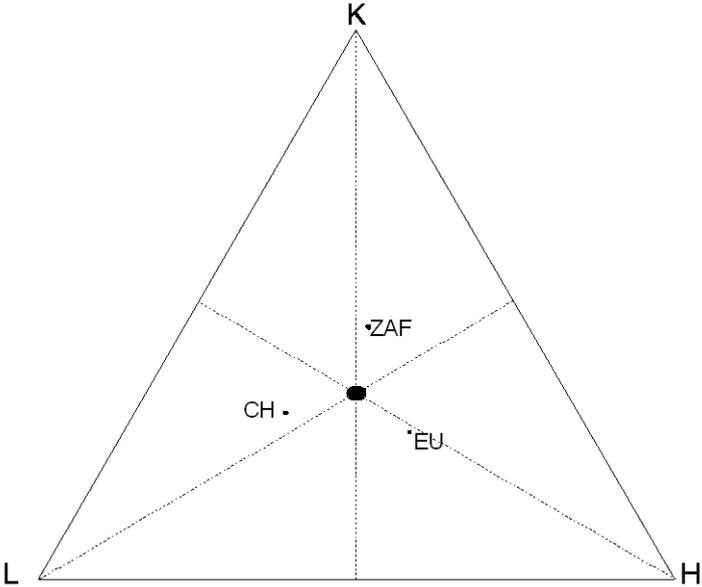
Figure 6: Model specific endowment triangle

The next section will use the same concept of the endowment triangle to include the relative factor intensities of the industries into the model.

Production technology and the triangles of diversification

To analyse the relative factor intensities within an industry the industry input vectors, i.e. the relative ratios of the three production factors used in one industry, are plotted in the endowment triangle. It should be clear from the theoretical discussion of the model, that the exact position of the input vectors is determined by the factor prices. But before going into the model specifics, the notion of endowment triangles must be discussed. Assume that the input vectors of three industries are known then these vectors can be drawn in the endowment triangle. As pointed out by Leamer (1987) it is now possible to divide the endowment triangle into triangles by connecting the vertices of the triangle and the input vectors with each other. The resulting triangles are called the triangles of diversification. Although there are alternative ways to establish the triangles of diversification, due to the influence of good and factor prices on the procedure of linear programming, this is of no further relevance for this paper as the discussion of Figure 7 will show.
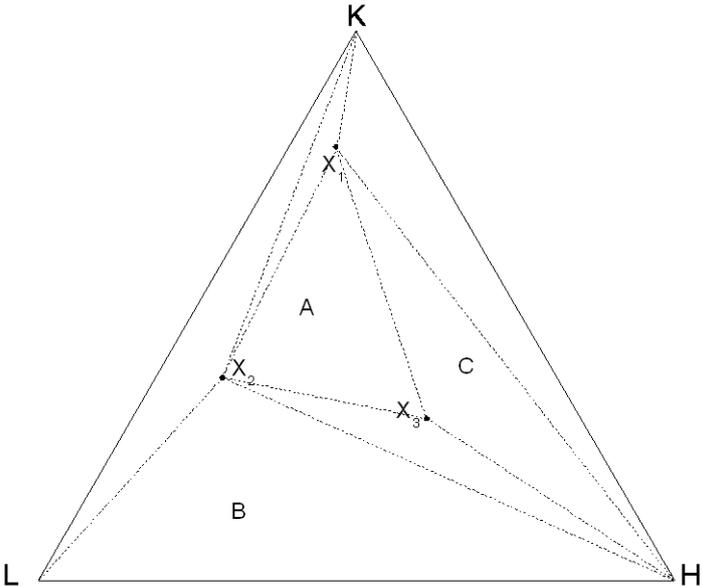
Figure 7: Triangles of diversification

Figure 7 shows one possible set of triangles of diversification based upon three industry input vectors (X_1, X_2 and X_3)[5]. Notwithstanding the fact that the endowment triangle comprises seven different triangles of diversification, it is sufficient to discuss only three of them. If a country´s endowment vector lies within the diversification triangle A, no factor is redundant

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[5] Note that all industries make use of all factors.

and all three industries produce a positive amount of their respective commodity. Countries with an endowment vector in region B produce only goods from industry 2 and both skilled and less-skilled labour are redundant. Countries in triangle C produce goods from industry 1 and industry 2 and only skilled labour is a redundant production factor. For this paper only region A is of further interest, because the assumption was made that each country produces goods from each industry. Moreover, although different ways are thinkable to divide the endowment triangle in triangles of diversification, triangle A will always remain the same. Concerning the factor returns one should note that within each triangle of diversification the remuneration of factors will be the same, whereas between different triangles differences will occur.

The above described characteristics of the triangles of diversification result from the application of linear programming on the model. By means of linear programming the following function can be maximised:

$$p_1 X_1 + p_2 X_2 + p_3 X_3.$$

This maximising problem is subject of constraints, already discussed in the model:

$$k_1 X_1 + k_2 X_2 + k_3 X_3 \leq \overline{K},$$
$$l_1 X_1 + l_2 X_2 + l_3 X_3 \leq \overline{L},$$
$$h_1 X_1 + h_2 X_2 + h_3 X_3 \leq \overline{H},$$
$$X_i \geq 0.$$

Using the simplex algorithm theory these constraints can be rewritten as equations:

$$\begin{pmatrix} k_1 & k_2 & k_3 & s_1 & 0 & 0 \\ l_1 & l_2 & l_3 & 0 & s_2 & 0 \\ h_1 & h_2 & h_3 & 0 & 0 & s_3 \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{pmatrix} = \begin{pmatrix} \overline{K} \\ \overline{L} \\ \overline{H} \end{pmatrix}, \text{ or in matrix notion,}$$

$$C \cdot X = V.$$

The variables s_1, s_2 and s_3 are slack variables, i.e. they represent the input vectors of some activities that do not produce any output. Therefore X_4, X_5 and X_6 represent slack factors, i.e. factors that are unemployed.

A basic solution of this problem can be written as:

$$X^B = B^{-1} \cdot V.$$

Moreover, the B matrix is a 3x3 dimensional matrix with elements derived from the C matrix. Values of X, i.e. factors, are zero, if their corresponding column in the C matrix is not used to establish the B matrix. If an optimal solution for a given endowment point combines thus the first, the second-last and the last columns of the C matrix to extract the B matrix, then only products of industry 1 will be produced and less-skilled as well as skilled labour will be partly unemployed. Another country for which the endowment point has an optimal solution that includes the first three columns of the C matrix will have no abundant factors and will produce the three different goods. Based upon the optimal solution of the linear programming problem it is possible to divide the endowment triangle into triangles of diversification. Note that the slack factors are represented by the vertices of the endowment triangle. Furthermore, endowment points in the same triangle of diversification have the same optimal solution, i.e. the solution involves the same activities.

Within this framework of the diversification triangle, as stated in the above section, the assumed factor intensities can be integrated. Based upon these assumptions, the factor intensities of the three industries can be written as:

<u>Industry 1</u>	<u>Industry 2</u>	<u>Industry 3</u>
$z_{1K} > 1 > z_{1L} > z_{1H}$	$z_{2L} > 1 > z_{2H} > z_{2K}$	$z_{3H} > 1 > z_{3K} > z_{3L}$
$\lambda_{1K} > \lambda_{1L} > \lambda_{1H}$	$\lambda_{2L} > \lambda_{2H} > \lambda_{2K}$	$\lambda_{3H} > \lambda_{3K} > \lambda_{3L}$
$\dfrac{K_1}{L_1} > \left(\dfrac{K}{L}\right)^W$	$\dfrac{L_2}{H_2} > \left(\dfrac{L}{H}\right)^W$	$\dfrac{H_3}{K_3} > \left(\dfrac{H}{K}\right)^W$
$\dfrac{K_1}{H_1} > \left(\dfrac{K}{H}\right)^W$	$\dfrac{L_2}{K_2} > \left(\dfrac{L}{K}\right)^W$	$\dfrac{H_3}{L_3} > \left(\dfrac{H}{L}\right)^W$
$\dfrac{L_1}{H_1} > \left(\dfrac{L}{H}\right)^W$	$\dfrac{H_2}{K_2} > \left(\dfrac{H}{K}\right)^W$	$\dfrac{K_3}{L_3} > \left(\dfrac{K}{L}\right)^W$

This information is enough to know that the input vectors of industry 1, 2 and 3 lie in the regions 6, 4 and 2 of Figure 5 respectively. This information can now be included into the further analysis of trade flows between the three countries.

Trade flows between open economies

Figure 8 combines the endowment triangle with the diversification triangle A from the above section. Due to the assumption that in each country the output of each industry is positive, the diversification triangle comprises the endowment points of all countries. Thus, it is also known that the remuneration of one factor is the same across countries, which also implies that all individuals face the same commodity prices. These results, combined with the assumption of homothetic consumer taste, mean that consumption at the world endowment point is representative for the three countries and by comparison it is possible to define exports and imports.
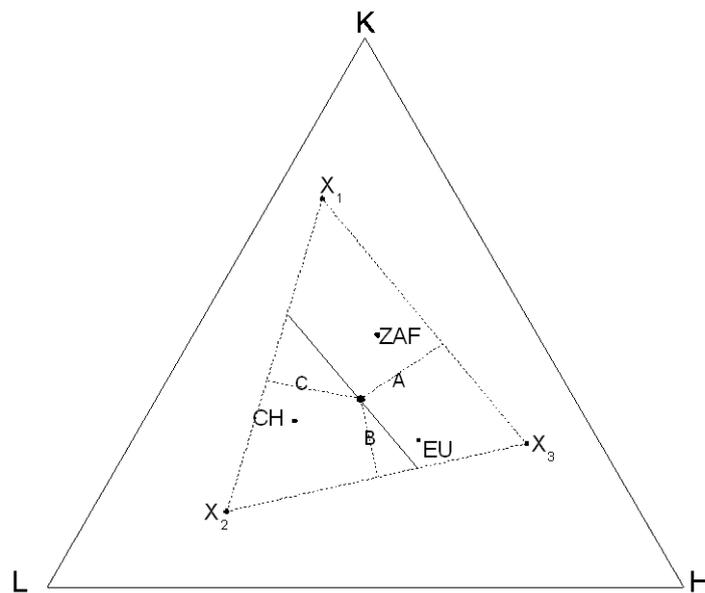
Figure 8: A 3x3x3 trade model

In Figure 8 three lines A, B and C emanate from the world endowment point. Each of these lines describes the shortest distance between the endowment point and one leg of the diversification triangle. These distances are indicators for the consumption of the opposite good, e.g. line A is an indicator for the consumption of good 3, and can be compared with other distance lines (e.g. B or C) to get a notion of the relative consumption. These "barycentric coordinates" (Leamer 1987: 973) can also be interpreted as indicators for world production, because the world is a closed system where production equals consumption. This allows for comparison of the barycentric coordinates of the world with those from each individual country. By doing so exports and imports can uniquely be defined. For industry 3 the world production is given by the distance A, which is also the distance between a line that

runs parallel with the X_1-X_3-line through the world endowment point and the H-K-leg. Countries between these two lines produce relatively less of good 2 and therefore import this good, whereas countries with an endowment point closer to X_2 export this good. Under the made assumptions, it should be clear that South Africa and the EU are importers of commodity X_2, the good with a relative less-skilled labour intensive production, and China would be the supplier of this product. Furthermore South Africa and China import the skilled labour intensive good from the EU, and South Africa exports the goods that use the capital linked to mining and closely linked activities intensively to both the EU and China.

Concluding remarks

In general, the examination of the South African trade structure over time with respect to the EU and China made it clear that South Africa was mainly a supplier of minerals, metals and other natural resources. Simultaneously, it imported chemicals and capital goods from the EU and consumer goods such as textiles and telecommunication equipment from China. During the observed period specialisation was the dominant trend within the South African trade structure and this resulted in the low level of intra-industry trade.

From a theoretic point of view, it was rather clear that this trade pattern could best be described by means of an Heckscher-Ohlin-Samuelson trade model. The theoretical basis for this model was based upon earlier literature concerning multi-dimensional trade models and was combined with the concept of the endowment triangle. This approach allowed for a analysis of the trade flows between the considered countries.

As the attentive reader will have noticed, there are still some problems to be solved. First of all, the input vectors of production, which are used to draw the diversification triangle, arrive actually out of the blue, i.e. without any further discussion. The exact position of these vectors, however, still needs more theoretic justification. Another topic for further research is the change of output within an economy as it changes its autarkic regime into free trade, i.e. comparative statics.

The title of this paper refers to the fact that by means of the endowment triangle it is possible to make statements about trade flows, representing the dial of a clock. At the same time these

trade flows result from the application of a general equilibrium model, in which all components of an economy are tightly intertwined and influence each other. If one component is changed, it will influence the overall equilibrium. These multidimensional and reciprocal interactions can remind someone of the wheels of a (mechanical) clockwork.

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