OPTIMAL MONETARY AND FISCAL POLICY IN THE EMU: DOES FISCAL POLICY COORDINATION MATTER?*

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Abstract

I develop a DSGE model to revise the question of how to conduct monetary and fiscal policy in a currency union. In contrast with the recent literature which assumes coordination, this paper analyzes the case of no-coordination either among fiscal authorities or between fiscal and monetary authorities. I show that the normative prescriptions emphasized by previous analyses are not valid any more once policymakers are not coordinated. Indeed, in this case on the hand, the common central bank should take into account the distortions caused by the independent fiscal authorities. As a consequence, it is not optimal to stabilize the average union inflation as if the currrency union were a closed economy. On the other hand, if there isn't a common agreement to coordinate fiscal policies, autonomous government should use the government expenditure as a stabilization tool even if shocks are symmetric.

Keywords: Monetary and Fiscal Policy, Policy Coordination, Terms of Trade.

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1 Introduction

The birth of the European Monetary Union (EMU) has sparkled interest in the question of how to conduct monetary and fiscal policy for a group of countries that share the same currency. There is a growing body of research that has tried to assess this issue within a fully micro-founded dynamic general equilibrium framework. However this literature relies on the existence of a supra-national authority to which all monetary and fiscal policy decisions have been delegated. Yet, as matter of fact, in the EMU only the monetary policy is under the control of a common authority, the European Central Bank (ECB), whereas, even if bound by the Stability and Growth Pact (SGP), fiscal policies are still decided at national level. Consequently the following questions arise: How should monetary and fiscal policies be conducted in a monetary union where there is a common central bank but autonomous fiscal policies? Does this institutional arrangement lead to different normative prescriptions with respect to those highlighted by the previous literature?

In order to answer such questions, this paper uses a generalized version of the DSGE model laid out by Galí and Monacelli (2008)[1] and compares two different policy regimes: the regime of fiscal policy coordination considered as a benchmark, already analyzed by Galí and Monacelli (2008) themselves, Beetsma and Jensen (2004) and Beetsma and Jensen (2005)[2] and the regime of fiscal policy no-coordination.

In our basic setup, the world is framed as a continuum of small open economies. Each country government chooses the optimal provision of a public bundle and sets a time-invariant labor subsidy. The presence of lump sum taxes ensures the compliance with SGP limits and rules out the additional problem of choosing how to finance optimally the public expenditure. Within this framework, under fiscal policy coordination, monetary and fiscal policies are chosen by a common policymaker in order to maximize the average union welfare. Conversely, under fiscal policy no-coordination[3], there is a multiplicity of policy authorities each of those takes as given other policymakers' decisions: governments that are concerned only about the welfare of their own country and the central bank of the Monetary Union that has the maximization of the average union welfare as objective.

According to the results, the no-coordination among fiscal authorities matters for the design of both optimal monetary and fiscal policies. The driving force of this finding

[1]See also Galí and Monacelli (2005). Differently from Galí and Monacelli (2008), not only final private goods but even public goods and intermediate inputs are traded, while the elasticity of substitution between home and foreign goods is not restricted to be equal to one. Moreover in the preference specification the intertemporal elasticities of substitution of public and private consumption are not necessarily equal. As it will be clarified below, the first two generalizations strengthen the incentive of uncoordinated fiscal policymakers to generate aggregate distortions. Conversely the last assumption is crucial to explain the results in the case of shocks to technology.

[2]Even Ferrero (2005) contributed to this debate. He analyzed the case coordination in which, however, the exogenous government expenditure is financed thought distortionary taxes and a riskless bonds.

[3]There are some old contributions that consider the case of no-coordination (for instance Lambertini and Dixit (2003)). However, in general these papers do not assume fully-micro-founded welfare criteria. An exception in this respect is the work by Lombardo and Sutherland (2004) which though treat only marginally the case of a monetary union and reach results opposite to those of this paper by assuming an efficient steady state and considering only the case of optimal simple rules.

stems from countries monopoly power on their terms of trade. Indeed, given the imperfect substitutability between bundles produced in different countries, uncoordinated policymakers have an incentive to try to influence the terms of trade in their favor. This incentive works both at the steady state and over the business cycle[4]. At the steady state, independent fiscal authorities act as a monopolist. They try to increase the demand of the home produced good and to decrease its supply by over-expanding the government expenditure and reducing the labor subsidy. In this way they seek to render home produced goods more expensive in order to externalize to other countries' consumers the costs of production and taxation. Over the business cycle instead they use the government expenditure to restrain the terms of trade volatility and hence reduce the cost of the volatility of output or private consumption at other countries' expense.

This mechanism explains the differences in the policy prescriptions under coordination and no-coordination. Under the benchmark case of fiscal policy coordination, Galí and Monacelli (2008), Beetsma and Jensen (2004) and Beetsma and Jensen (2005) have pointed out two main findings. First, under the optimal policy mix, the common monetary policy should seek to stabilize the average union inflation following the same normative prescriptions valid in a closed economy. Therefore, under technology shocks, it should pursue the stability of the average union price level; under markup shocks, it has to trade off between stabilizing the average inflation and the average output gap. Secondly, in a monetary union fiscal policy is a useful tool for macroeconomic stabilization of single country economies. Indeed, at single country level fiscal policy should be employed to stabilize the effects of the idiosyncratic shocks given that, because of the adoption of the common currency, the central bank is able only to stabilize the aggregate economy. However, at the aggregate level fiscal policy should only ensure on average the efficient provision of the public goods.

Under fiscal policy no-coordination, the previous results no longer hold. With regards to monetary policy, the common central bank should cope with the aggregate distortions generated both at the steady state and over the business cycle by independent governments and shouldn't stabilize the average union economy as if it were a closed economy. Therefore in general, in the presence of productivity shocks strict inflation targeting is not optimal. Indeed, under flexible prices output volatility is inefficiently high for the at least two reasons. On the one hand national authorities have an incentive to manipulate the terms of trade to their own advantage even over the business cycle. On the other hand the steady state government expenditure share in output is inefficiently high and thus amplifies the effects of government expenditure shocks on output fluctuations[5]. Moreover in the response to markup shocks, the monetary authority should be much more aggressive in fighting inflation under no-coordination than under coordination. This finding is explained by the inefficiently low steady state level of the output. Given that distortion, an increase in output volatility in response to

[4]...as pointed out by the previous literature: see, among others, Corsetti and Pesenti (2001), Benigno and Benigno (2003) and Epifani and Gancia (2008).

[5]...at least under the baseline calibration. Given the inefficiently high steady state government expenditure share in output, one percentage increase in the government expenditure expands more output under no-coordination than under coordination. Galí (1994) has already emphasized that the government's size may have an effect on output volatility.

markup shocks has some beneficial effects because it makes consumers willing to work more on average driving the economy towards the efficient allocation.

With regards to fiscal policies, given their incentives, independent governments do not ensure on average the efficient provision of the public goods. And in the case of markup shocks they are using government expenditure for stabilization purposes even if shocks are symmetric. Indeed, by taking as given what other policymakers are doing, they do not realize that the common central bank is already stabilizing the aggregate economy and they seek, on their own, to stabilize the undesirable effects of markup shocks.

The paper is organized as follows. Section 2 describes the basic framework. Section 3 introduces the equilibrium conditions. Section 4 examines the case of full coordination. Section 5 the case of no-coordination. Section 6 concludes.

2 The model

The world consists of a continuum of small open economies[6]. In each country there are two sectors: a competitive sector that produces one final good by using both home and foreign country intermediate inputs; a monopolistic competitive sector that produces a continuum of intermediate differentiated goods by using as input labor which is assumed immobile across countries.

2.1 Preferences

Preferences of a generic country representative household are defined over a private consumption bundle, C_t, a public consumption bundle, G_t and hours of labor $N_t(h)$[7]:

$$\sum_{t=0}^{\infty}\beta^t E_0\left[\frac{C_t^{1-\sigma}}{1-\sigma}+\chi\frac{G_t^{1-\gamma}}{1-\gamma}-\frac{N_t(h)^{\varphi+1}}{\varphi+1}\right] \quad 0<\beta<1 \tag{1}$$

where, as usual, β stands for the intertemporal preferences discount factor and χ is the weight attached to public consumption. Agents consume all the goods produced in the world economy. However preferences exhibit home bias. The private consumption index is, in fact, a CES aggregation of the following type:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad \eta>0 \tag{2}$$

with $1-\alpha$ being the degree of home bias in the private consumption and η denoting the elasticity of substitution between $C_{H,t}$, and $C_{F,t}$. $C_{H,t}$ represents the home household's consumption of the single home final good while $C_{F,t}$ is a CES aggregation of the goods produced in foreign countries namely:

$$C_{F,t} \equiv \left[\int_0^1 C_{j,t}^{\frac{\eta-1}{\eta}}\,dj\right]^{\frac{\eta}{\eta-1}} \tag{3}$$

[6]The general framework draws on Galí and Monacelli (2005) and Galí and Monacelli (2008).
[7]In this and in the following subsections we abstract from indexing the small open economy of reference.

η then represents even the elasticity of substitution between different foreign goods.

The public bundle is defined similarly to the private bundle, that is:

$$G_t \equiv \left[(1-\nu)^{\frac{1}{\eta}} G_{H,t}^{\frac{\eta-1}{\eta}} + \nu^{\frac{1}{\eta}} G_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad \eta > 0 \tag{4}$$

with

$$G_{F,t} \equiv \left[\int_0^1 G_{j,t}^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \tag{5}$$

where $1-\nu$ indicates the degree of home bias in the public consumption which, in general, is allowed to be different from $1-\alpha$[8].

Public and private consumption index definitions (2), (4), (3) and (5) allow to determine consistent definitions of price indexes[9]. In particular, $P_{C,t}$ and $P_{G,t}$, the private and the public consumers' price indexes[10] are given by:

$$P_{C,t} \equiv \left[(1-\alpha)P_t^{1-\eta} + \alpha P_t^{*1-\eta}\right]^{\frac{1}{1-\eta}} \tag{6}$$

$$P_{G,t} \equiv \left[(1-\nu)P_t^{1-\eta} + \nu P_t^{*1-\eta}\right]^{\frac{1}{1-\eta}} \tag{7}$$

with P_t^* being specified as:

$$P_t^* \equiv \left[\int_0^1 P_t^{j\,1-\eta} dj\right]^{\frac{1}{1-\eta}} \tag{8}$$

Thus P_t and P_t^j are producers' price indexes [11] There are no trading frictions being the law of one price assumed to hold in all single good markets. However, given the home biased preferences, the purchasing power parity does not hold for indexes $P_{C,t}$ and $P_{G,t}$.

2.2 Consumption demand, portfolio choices and labor supply

The consumption and price index definitions allow to solve the consumer problem in three stages. In the first two stages, agents decide how much real net income to allocate to buy goods produced at home and abroad. According to the set of optimal conditions, it is possible to determine agent demands for $C_{H,t}$, $C_{F,t}$ and $C_{j,t}$, as:

$$C_{H,t} = (1-\alpha)\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t \qquad C_{F,t} = \alpha\left(\frac{P_t^*}{P_{C,t}}\right)^{-\eta} C_t \qquad C_{j,t} = \left(\frac{P_t^j}{P_t^*}\right)^{-\eta} C_{F,t} \tag{9}$$

[8]In fact Brülhart and Trionfetti (2004) point out that the home bias of public goods is higher than home bias of private goods.

[9]Namely price and consumption indexes are such that at the optimum expenditures for total consumption of both private and public goods, $P_t C_{H,t} + \int_0^1 P_t^j C_{j,t} dj$ and $P_t G_{H,t} + \int_0^1 P_t^j G_{j,t} dj$ are equal respectively to $P_{C,t}C_t$ and $P_{G,t}G_t$.

[10]In what follows, CPI stands for consumers' price index.

[11]Again in what follows, PPI stands for producers' price index.

for all j. The third stage coincides with the standard consumer problem. Agents are monopolistic competitive labor suppliers. Thus they maximize (1) with respect to C_t, D_{t+1} and $N_t(h)$ subject to the following sequence of constraints:

$$E_t\{Q_{t,t+1}D_{t+1}\} = D_t + W_t(h)N_t(h) - P_{C,t}C_t + T_t \tag{10}$$

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\upsilon_t} N_t \tag{11}$$

where:

$$W_t \equiv \left[\int_0^1 W_t(h)^{1-\upsilon_t} dh\right]^{\frac{1}{1-\upsilon_t}} \tag{12}$$

Constraint (10) is the budget constraints which states that nominal saving, net of lump sum transfers has to equalize the nominal value of a state contingent portfolio. In fact $W_t(h)$ stands for the per hour nominal wage, $Q_{t,t+1}$ denotes what is usually called the stochastic discount factor and D_{t+1} is the payoff of one maturity portfolio that includes firm shares.

Constraint (11) is a consequence of a CES aggregation of labor inputs which will be specified in the next sub-section and implicitly assumes that the elasticity of demand of labor, υ_t, is time-varying but equal across agents as in Clarida, Galí and Gertler (2002). Finally (12) is simply the aggregate wage index. Domestic and international markets are assumed to be complete.

By the optimality conditions of the household problem:

$$(1 + \mu_t)N_t(h)^{\varphi}C_t^{\sigma} = \frac{W_t}{P_{C,t}} \tag{13}$$

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_{C,t}}{P_{C,t+1}}\right) = Q_{t,t+1} \tag{14}$$

which hold in all states of nature and at all periods and where $\mu_t \equiv \frac{1}{\upsilon_t - 1}$.

According to (13), workers set the real wage as markup over the marginal rate of substitution between consumption and leisure, while the value of the intertemporal marginal rate of substitution of consumption should equalize the stochastic discount factor. Notice that since wages are perfectly flexible $N_t(h)$=N_t and $W_t(h)$=W_t for all h and t.

2.3 Final good aggregate demand

In each country the demand for the final good is the sum of four components: the demands of domestic and foreign households and governments namely:

$$Y_t = C_{H,t} + \int_0^1 C_{H,t}^j dj + G_{H,t} + \int_0^1 G_{H,t}^j dj \tag{15}$$

Condition (15) can be rewritten as:

$$Y_t = (1-\alpha)\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_t}{P^j_{C,t}}\right)^{-\eta} C^j_t dj + (1-\nu)\left(\frac{P_t}{P_{G,t}}\right)^{-\eta} G_t + \nu \int_0^1 \left(\frac{P_t}{P^j_{G,t}}\right)^{-\eta} G^j_t dj \tag{16}$$

which follows from equation (3)[12] and the fact that:

$$G_{H,t} = (1-\nu)\left(\frac{P_t}{P_{G,t}}\right)^{-\eta} G_t \qquad G_{F,t} = \nu\left(\frac{P^*_t}{P_{G,t}}\right)^{-\eta} G_t \qquad G_{j,t} = \left(\frac{P^j_t}{P^*_t}\right)^{-\eta} G_{F,t} \tag{17}$$

for all j. According to (17) independently of the aggregate level of G_t, governments choose good demands by minimizing the total expenditure $P_t G_{H,t} + \int_0^1 P^j_t G_{j,t} dj$.

2.4 Firms and technology in the final good sector

Each final good is produced by using both home and foreign inputs according to the following CES technology:

$$Y_t = \left[(1-\psi)^{\frac{1}{\eta}} \left(Y^I_{H,t}\right)^{\frac{\eta-1}{\eta}} + \psi^{\frac{1}{\eta}} \left(Y^I_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad \eta > 0 \tag{18}$$

where $1-\psi$ is the degree of home bias in intermediate goods. Y^I_H and Y^I_F are defined as:

$$Y^I_{H,t} \equiv \left[\int_0^1 \left(y^I_{H,t}(k)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad Y^I_{F,t} \equiv \left[\int_0^1 \left(Y^I_{j,t}\right)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \tag{19}$$

with $Y^I_{j,t} \equiv \left[\int_0^1 \left(y^I_{j,t}(k)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}$ for all j and $y^I_{H,t}(k)$ and $y^I_{j,t}(k)$ being the demands for the k type of intermediate good produced in the home country and in country j respectively.

The final sector is perfectly competitive. Therefore firms maximize profits taking P_t, the price of the final good, as given. The optimality conditions of this problem lead to the following single and aggregate input demands:

$$y^I_{H,t}(k) = \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} Y^I_{H,t} \qquad y^I_{j,t}(k) = \left(\frac{p_{j,t}(k)}{P_{j,t}}\right)^{-\varepsilon} Y^I_{j,t} \tag{20}$$

$$Y^I_{H,t} = (1-\psi)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} Y_t \qquad Y^I_{F,t} = \psi\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} Y_t \qquad Y^I_{j,t} = \left(\frac{P_{j,t}}{P_{F,t}}\right)^{-\eta} Y^I_{F,t} \tag{21}$$

which allow to determine consistently the price indexes for final and intermediate goods as:

$$P_t = \left[(1-\psi)\left(P_{H,t}\right)^{1-\eta} + \psi\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{22}$$

[12]... with the symmetric equations for foreign countries.

$$P_{H,t} = \left[\int_0^1 p_{H,t}(k)^{1-\varepsilon} dk \right]^{\frac{1}{1-\varepsilon}} \qquad P_{F,t} = \left[\int_0^1 (P_{j,t})^{1-\eta}\, dj \right]^{\frac{1}{1-\eta}} \qquad P_{j,t} = \left[\int_0^1 p_{j,t}(k)^{1-\varepsilon} dk \right]^{\frac{1}{1-\varepsilon}} \tag{23}$$

where $p_{j,t}(k)$ is the price of intermediate input k produced in country j.

2.5 Intermediate good aggregate demand

The demand for home intermediate goods is generated by the demands of both home and foreign final good producers, namely:

$$y_{H,t}(k) \equiv y_{H,t}^{I}(k) + \int_0^1 y_{H,t}^{I,j}(k) dj \tag{24}$$

Condition (24) can be rewritten as:

$$y_{H,t}(k) = \left(\frac{p_{H,t}(k)}{P_{H,t}} \right)^{-\varepsilon} \left[(1-\psi) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} Y_t + \psi \int_0^1 \left(\frac{P_{H,t}}{P_t^j} \right)^{-\eta} Y_t^j dj \right] \tag{25}$$

which follows from equations (20) and (21)[13]. Given (25) it is possible to recover the aggregate demand $Y_{H,t} \equiv \left(\int_0^1 (y_{H,t}(k))^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}$. In fact by properly integrating (25) we obtain:

$$Y_{H,t} = \left[(1-\psi) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} Y_t + \psi \int_0^1 \left(\frac{P_{H,t}}{P_t^j} \right)^{-\eta} Y_t^j dj \right] \tag{26}$$

2.6 Firm technology and price setting in the intermediate good sector

In the intermediate sector each firm produces a single differentiated good with a constant return to scale technology of the type:

$$y_{H,t}(k) = A_t N_t(k) \tag{27}$$

with $N_t(k) = \left[\int_0^1 N_t(h)^{\frac{\upsilon_t - 1}{\upsilon_t}} dh \right]^{\frac{\upsilon_t}{\upsilon_t - 1}}$ and being the labor input and A_t the specific country technology shock. Given (27) and the fact that $N_t = N_t(h)$ for all h, the aggregate relationship between output and labor can be read as:

$$N_t = \frac{Y_{H,t}}{A_t} Z_t \tag{28}$$

where $Z_t \equiv \int_0^1 \frac{y_{H,t}(k)}{Y_{H,t}} dk$, and $N_t \equiv \int_0^1 N_t(k) dk$. Given (24) and (26) then $Z_t \equiv \int_0^1 \left(\frac{p_{H,t}(k)}{P_{H,t}} \right)^{-\varepsilon} dk$; thus Z_t can be interpreted as an index of the relative price dispersion across firms. We assume that good prices adjust according to a staggered mechanism

[13]... with the symmetric equations for foreign countries.

à la Calvo. Therefore in each period a given firm can reoptimize its price only with probability $1-\theta$. As a result the fraction of firms that set a new price is fixed and the aggregate producer price index of the intermediate goods evolves accordingly to:

$$P_{H,t}^{(1-\varepsilon)} = \theta P_{H,t-1}^{(1-\varepsilon)} + (1-\theta)\tilde{p}_{H,t}(k)^{(1-\varepsilon)} \tag{29}$$

with $\tilde{p}_{H,t}(k)$ being the optimal price. Firms maximize the discounted expected sum of the future profits that would be collected if the optimal price could not be changed namely:

$$\sum_{s=0}^{\infty} (\theta)^s E_t \left\{ Q_{t,t+s} y_{H,t+s}(k) \left[\tilde{p}_{H,t}(k) - MC_{t+s}^n \right] \right\} \tag{30}$$

where $y_{H,t}(k)$ is given by (25) and $MC_t^n = \frac{(1-\tau)W_t}{A_t}$ is the nominal marginal cost with τ indicating a labor subsidy distributed to firms by the fiscal authority which is not supposed to vary over the business cycle. Taking into account (14) and that $MC_t \equiv \frac{MC_t^n}{P_{H,t}}$, the optimality condition of the firm problem can be written as:

$$\sum_{s=0}^{\infty} (\beta\theta)^s E_t \left\{ C_{t+s}^{-\sigma} \left(\frac{\tilde{p}_{H,t}(k)}{P_{H,t+s}} \right)^{-\varepsilon} Y_{H,t+s} \frac{P_{H,t}}{P_{C,t+s}} \left[\frac{\tilde{p}_{H,t}(k)}{P_{H,t}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{H,t+s}}{P_{H,t}} MC_{t+s} \right] \right\} = 0 \tag{31}$$

Condition (31) states implicitly that firms reset their prices as a markup over a weighted average of the current and expected marginal costs, where the weight of the expected marginal cost at some date $t+s$ depends on the probability that the price is still effective at that date.

3 Equilibrium

The purpose of this section is twofold: on the one hand to recover the full set of conditions necessary and sufficient to determine the equilibrium of the monetary union; on the other hand to rewrite the single country equilibrium conditions in terms only of single country and average union variables. Indeed in this way, it is possible to simplify the fiscal policy problem under no-coordination. Being infinitesimally small, single country behaviour does not affect the average union performance. As a consequence, under no-coordination, the fiscal policy problem can be formulated just considering single country (and not the full set of the monetary union) equilibrium conditions.

3.1 International risk sharing

Under complete markets[14], condition (14) and the corresponding conditions for other countries imply:

$$\frac{P_{C,t}^j}{P_{C,t}} = \left(\frac{C_t^j}{C_t} \right)^{-\sigma} \tag{32}$$

[14]...and the assumption that the state contingent wealth at time zero is such that the lifetime discounted budget constraints are identical across agents.

for all t and j.

Notice that $P_t^* = \left[\int_0^1 (P_{C,t}^j)^{1-\eta} dj\right]^{\frac{1}{1-\eta}}$ and let:

$$C_t^* \equiv \left[\int_0^1 (C_t^j)^{\sigma(\eta-1)} dj\right]^{\frac{1}{\sigma(\eta-1)}} \tag{33}$$

Hence by properly integrating (32) we obtain:

$$\frac{P_t^*}{P_{C,t}} = \left(\frac{C_t^*}{C_t}\right)^{-\sigma} \tag{34}$$

According to equation (32) and its aggregate version (34), when financial markets are complete, the marginal rate of substitution between home and other country consumption (or the average union consumption) has to be equal to the corresponding relative price. Thus when there is increase in the home relative to foreign CPI, domestic households decrease consumption relative to foreigners. Indeed the terms of trade improvement in the home country[15] - associated with the relative increase in the CPI - induces private agents to reallocate the consumption between home and foreign goods. Then, because of the home bias, the home country consumers would decrease the *total* private consumption more than foreigners [16].

By combining (34) with (6), (7) and (22) and considering that $P_t^* = P_{F,t}$, it follows that:

$$\frac{P_t}{P_{C,t}} = \left[\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\left(\frac{C_t^*}{C_t}\right)^{-\sigma(1-\eta)}\right]^{\frac{1}{1-\eta}} \tag{35}$$

$$\frac{P_{G,t}}{P_{C,t}} = \left[(1-\nu)\frac{P_t}{P_{C,t}}^{1-\eta} + \nu\left(\frac{C_t^*}{C_t}\right)^{-\sigma(1-\eta)}\right]^{\frac{1}{1-\eta}} \tag{36}$$

$$\frac{P_{H,t}}{P_t} = \left[\frac{1}{1-\psi} - \frac{\psi}{1-\psi}\left(\frac{C_t^*}{C_t}\right)^{-\sigma(1-\eta)}\left(\frac{P_t}{P_{C,t}}\right)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{37}$$

which say that all the single country relative prices $P_t/P_{C,t}$, $P_{G,t}/P_{C,t}$ and $P_{H,t}/P_t$ and the terms of trade P_t^*/P_t and $P_t^*/P_{H,t}$ [17] are function exclusively of the difference between single country and average union private consumption.

In addition given (14) and (34):

$$\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \Pi_{t+1}^{*-1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \Pi_{C,t+1}^{-1} \tag{38}$$

$$= \left(\frac{P_t}{P_{C,t}}\right)\left(\frac{P_{Ht}}{P_{C,t}}\right)\Pi_{H,t+1}^{-1}\left(\frac{P_t}{P_{C,t}}\right)^{-1}\left(\frac{P_{H,t}}{P_{C,t}}\right)^{-1} \tag{39}$$

[15]Namely the prices of the foreign goods in terms of home goods, that is P_t^*/P_t and $P_t^*/P_{H,t}$.

[16]In fact because of the home bias, even if there are complete markets, private agents consumption is not equal across countries.

[17]In fact by (6) and (22), $P_{C,t}/P_t = \left[(1-\alpha) + \alpha\,(P_t^*/P_t)^{(1-\eta)}\right]^{\frac{1}{1-\eta}}$ and $P_t/P_{H,t} = \left[(1-\psi) + \psi\,(P_t^*/P_{H,t})^{(1-\eta)}\right]^{\frac{1}{1-\eta}}$. P_t^*/P_t and $P_t^*/P_{H,t}$ are the so called *effective* terms of trade. In what follows, unless specified differently, we will refer only to the effective terms of trade.

with $\Pi_t^* \equiv P_t^*/P_{t-1}^*$ and $\Pi_{C,t} \equiv P_{C,t}/P_{C,t-1}$.

Thus in equilibrium the value of intertemporal marginal rate of substitution of private consumption should be equal across countries. This last condition combined with (35) and (37) can be log-linearized as:

$$\pi_{H,t} - \pi_t^* = -\omega_4 \left(\Delta \hat{c}_t - \Delta \hat{c}_t^*\right) \tag{40}$$

where $\omega_4 \equiv \frac{\sigma}{(1-\alpha)(1-\psi)}$[18]. (38) and (40) relate consumption variations differential from the union average to the corresponding *domestic* inflation differential. Moreover under complete markets, from conditions (14) and (34) it follows:

$$\frac{1}{1+i_t^*} = E_t\{Q_{t,t+1}\} = \beta E_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left(\Pi_{t+1}^{*-1}\right) \right\} \tag{41}$$

When markets are complete, the price of a riskless portfolio should be equal to the price of the one-period riskless bond, being i_t^* its gross yield.

The log-linear approximation of (41) leads to:

$$\hat{c}_t^* = E_t\{\hat{c}_{t+1}^*\} - \frac{1}{\sigma}(i_t^* - E_t\{\pi_{t+1}^*\} - \varrho) \tag{42}$$

where $\varrho \equiv -log\beta$. Condition (42) is the so called IS curve that relates the average union intertemporal marginal rate of substitution of private consumption with the real interest rate.

By(38), (41) can be read as:

$$\frac{1}{1+i_t^*} = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\Pi_{C,t}^{-1}\right) \right\} \tag{43}$$

In other words by (38) condition (41) is satisfied even at the single country level[19]. Notice that outside a monetary union, where exchange rates are floating, (38) and (41) do not necessarily hold because the fluctuations of nominal exchange rates themselves assure the equality between the values of intertemporal marginal rate of substitution of private consumption across countries and give reason of differences in the nominal interest rates. For this reason we can interpret (38) as a constraint imposed by the adoption of a common currency. Indeed according to this condition in response to asymmetric shocks the terms of trade cannot adjust instantaneously because of the sluggish prices adjustment and the fixed exchange rates.

3.2 Good market clearing conditions

To rewrite the resource constraints as function of only aggregate variables, note that $P_t/P_{C,t}^j = (P_t/P_{C,t})(P_{C,t}/P_{C,t}^j)$. Similarly $P_t/P_{G,t}^j = (P_t/P_{C,t})\,(P_{C,t}/P_{C,t}^j)(P_{C,t}^j/P_{G,t}^j)$ and $P_{H,t}P_t^j = (P_{H,t}/P_t)(P_t/P_{C,t})(P_{C,t}/P_{C,t}^j)(P_{C,t}^j/P_{j,t})$. Then by substituting (32) in (16) and (26) we can express the good market clearing conditions as:

[18]Henceforth the following conventions are used: $\hat{x}_t$ stands for the log deviation of X_t from the symmetric zero inflation steady state while $\Delta\hat{x}_t \equiv \hat{x}_t - \hat{x}_{t-1}$ and $\hat{x}_t^* \equiv \int_0^1 \hat{x}_t^i di$.

[19]However (38) is not only a sufficient but also a necessary condition for (43) to be satisfied given (41).

$$Y_t = \left(\frac{P_t}{P_{C,t}}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha C_t^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}}{P_{G,t}}\right)^{-\eta} G_t + \nu C_t^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right] \tag{44}$$

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\psi)Y_t + \psi\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right] \tag{45}$$

where:

$$\Upsilon_{C,t} \equiv \left[\int_0^1 C_t^{j\,1-\sigma\eta} dj\right]^{\frac{1}{1-\sigma\eta}}$$

$$\Upsilon_{G,t} \equiv \left[\int_0^1 C_t^{j\,-\sigma\eta}\left(\frac{P^j_{C,t}}{P^j_{G,t}}\right)^{-\eta} G_t^j dj\right]^{\frac{1}{1-\sigma\eta}} \qquad \Upsilon_{Y,t} \equiv \left[\int_0^1 C_t^{j\,-\sigma\eta}\left(\frac{P^j_{C,t}}{P_{j,t}}\right)^{-\eta} Y_t^j dj\right]^{\frac{1}{1-\sigma\eta}} \tag{46}$$

Rewriting the good market clearing conditions in this way lead to the following consideration: any improvement in the domestic terms of trade makes private agents willing to switch expenditure from home to foreign goods[20]; however if isolated, this same improvement does not increase the demands for final and intermediate foreign goods because countries are assumed to be small.

The log-linear approximations of the resource constraints (44) and (45) and of conditions (35), (36) and (37) allow to retrieve the following condition:

$$\hat{y}_{H,t} + \frac{\psi}{1-\psi}(\hat{y}_{H,t} - \hat{y}_t^*) = \rho\hat{c}_t + \rho(\delta-1)(\hat{c}_t - \hat{c}_t^*) + (1-\rho)\hat{g}_t - (1-\rho)\nu(\hat{g}_t - \hat{g}_t^*) \tag{47}$$

where

$$\delta \equiv \delta_1 + \delta_2\frac{(1-\rho)}{\rho} + \delta_3\frac{1}{\rho}$$

$$\delta_1 \equiv (1-\alpha) + \xi\alpha(2-\alpha) \qquad \delta_2 \equiv \xi\nu(2-\nu) \qquad \delta_3 \equiv \frac{\xi\psi(2-\psi)}{(1-\psi)} \tag{48}$$

with $\rho \equiv \frac{C}{Y}$ and $\xi \equiv \eta\sigma/(1-\alpha)$.

3.3 The Phillips curve

Given condition (31) the optimal price is determined as:

$$\frac{\tilde{p}_{H,t}(k)}{P_{H,t}} = \frac{K_t}{F_t} \tag{49}$$

with:

$$K_t \equiv \sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[C_{t+s}^{-\sigma}Y_{H,t+s}\left(\frac{P_{H,t+s}}{P_{H,t}}\right)^{\varepsilon}\frac{P_{H,t+s}}{P_{C,t+s}}\frac{\varepsilon}{\varepsilon-1}MC_{t+s}\right] \tag{50}$$

[20]what in the literature is called the *switching expenditure* effect.

$$F_t \equiv \sum_{s=0}^{\infty} (\beta\theta)^s E_t \left[C_{t+s}^{-\sigma} Y_{H,t+s} \left(\frac{P_{H,t+s}}{P_{H,t}} \right)^{\varepsilon-1} \frac{P_{H,t+s}}{P_{C,t+s}} \right] \tag{51}$$

which can be read as:

$$K_t = C_t^{-\sigma} Y_{H,t} \frac{P_{H,t}}{P_{C,t}} \frac{\varepsilon}{\varepsilon-1} MC_t + \beta\theta E_t \left\{ \Pi_{H,t+s}^{\varepsilon} K_{t+1} \right\} \tag{52}$$

$$F_t = C_t^{-\sigma} Y_{H,t} \frac{P_{H,t}}{P_{C,t}} + \beta\theta E_t \left\{ \Pi_{H,t+1}^{\varepsilon-1} F_{t+1} \right\} \tag{53}$$

where:

$$MC_t = \frac{W_t}{P_{C,t}} \left(\frac{P_t}{P_{C,t}} \right)^{-1} \frac{1}{A_t} \tag{54}$$

Following Benigno and Woodford (2005), from (49) and (29) we can retrieve the next conditions:

$$\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta} = \left(\frac{F_t}{K_t} \right)^{\varepsilon-1} \tag{55}$$

$$Z_t = \theta Z_{t-1} \Pi_{H,t}^{\varepsilon} + (1-\theta) \left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{56}$$

which determines the law of motion of firms price dispersion. From the log linear approximation of (13) (50) (51) and (56):

$$\pi_{H,t} = \lambda \widehat{mc}_t + \beta E_t\{\pi_{H,t+1}\} \tag{57}$$

where:

$$\begin{aligned} \widehat{mc}_t \quad &= (\widehat{w}_t - \hat{p}_{c,t}) - (\widehat{p}_t - \hat{p}_{c,t}) - (\widehat{p}_{H,t} - \hat{p}_t) - \hat{a}_t \\ &= \varphi \hat{y}_{H,t} + \sigma \hat{c}_t + \omega_4((1-\psi)\alpha + \psi)(\hat{c}_t - \hat{c}_t^*) - (1+\varphi)\hat{a}_t + \hat{\mu}_t \end{aligned} \tag{58}$$

Condition (57) is the New Keynesian Phillips Curve direct consequence of the Calvo mechanism. As usual, current *domestic* inflation depends on the expectation on future *domestic* inflation and the current real marginal cost of producing intermediate goods. Being the economy open, in equilibrium that cost is determined by the real wage (which is equal to the marginal rate of substitution between consumption and leisure), the labour productivity and the relative prices of intermediate and final goods. These prices are determined as made clear by (35) and (37) by the differences of private consumption from the average.

The rational expectation stochastic equilibrium of the monetary union is then defined as the sequence of $\{C_t^i, Y_t^i, Y_{H,t}^i, \Pi_{H,t}^i, Z_t^i, F_t^i, K_t^i, \Pi_t^* \}_{t=0}^{\infty}$ for all i which, given $\{G_t^i\ , i_t^* \}_{t=0}^{\infty}$ for all i, τ and the initial condition Z_{-1}, satisfies, (39), (41), (44), (45), (52), (53), (55) and (56) for all i where $W_t^i/P_{C,t}^i$ $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$ and MC_t^i are determined according to (13), (35), (36), (37) and (54).

What it is still missing is to determine the optimal monetary and fiscal policies. This is the purpose of the next paragraphs.

4 The optimal policies

As mentioned in the introduction, the optimal monetary and fiscal policy mix is analysed under two different policy regimes: the regimes of coordination and no-coordination. Under coordination there is a common authority responsible for both monetary and fiscal policies which has the maximization of the average union welfare as objective. Under no-coordination there is a plurality of independent policymakers each of those takes other policy authorities' decisions as given. The central bank on the one hand which seeks to minimize the average losses of union households and the governments on the other hand which, conversely, are concerned about the average losses of the single country households. The solutions to the optimal policy problems under both regimes are derived by using the linear quadratic approach proposed by Benigno and Woodford (2005). This method requires to assume that policies are optimal from *timeless* perspective[21] and can be implemented as follows. First the zero-inflation deterministic steady state is retrieved; then a purely quadratic approximation to the single country and monetary union welfare around the deterministic steady state is obtained. Being the economies open and in the case of no-coordination the deterministic steady state distorted, these approximations are derived by using the second order approximations of the structural equations. Finally, given the purely quadratic approximations of policymakers' objectives, the optimal policies[22] are recovered by using as constraints the equilibrium conditions approximated up to the first order.

4.1 The case of coordination

Under coordination the optimal policy problem of the common authority can be formulated as the maximization of:

$$\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} + \chi \frac{G_t^{i\,1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1} \right] di \tag{59}$$

with respect to C_t^i, G_t^i, Y_t^i, $Y_{H,t}^i$, $\Pi_{H,t}^i$, Z_t^i, F_t^i and K_t^i for all i, subject to the equilibrium conditions (39), (44), (45), (52), (53), (55), and (56) for all i where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$ are determined according to (35), (36), (37). It is easy to show that the symmetric zero inflation deterministic steady state[23] reduces to the following system of equations:

$$C^{-\sigma} = Y^{\varphi} \tag{60}$$

$$\chi G^{-\gamma} = Y^{\varphi} \tag{61}$$

$$Y = C + G \tag{62}$$

[21] See also Benigno and Benigno (2006), Benigno and De Paoli (2005) and De-Paoli (2007).

[22] In the case of no-coordination, the Nash equilibrium policies are determined by the solutions to both the monetary and fiscal policy problems.

[23] See the appendix.

where $A = 1$. Conditions (60) and (61) equate the marginal rates of substitution (MRS) between private consumption and leisure and between public consumption and leisure to their marginal rates of transformation (MRT) while condition (62) is the resource constraint, that ensures the equilibrium between the demand for final goods and its relative supply. Thus, under coordination the steady state allocation is Pareto efficient. Notice that in order to implement this allocation, the common policymaker need two instruments: the government expenditure to provide an efficient level of public goods consistently with (60) , (61) and (62) and a labour subsidy to completely offset the monopolistic distortions of both labour and good markets.

The welfare approximation. As shown in the appendix, under coordination the average welfare of union households can be approximated as follows:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \Big[\frac{\varepsilon}{\lambda}(\pi^i_{H,t})^2 + \varphi(\tilde{y}^{c,i}_{H,t})^2 + \gamma(1-\rho)(\tilde{g}^{c,i}_t)^2 + \sigma\rho(\tilde{c}^{c,i}_t)^2$$
$$+2\varpi^c_1(\tilde{g}^{c,i}_t - \tilde{g}^{c,*}_t)(\tilde{c}^{c,i}_t - \tilde{c}^{c,*}_t) + \varpi^c_2(\tilde{c}^{c,i}_t - \tilde{c}^{c,*}_t)^2\Big] di + s.o.t.i.p. \tag{63}$$

with:

$\varpi^c_1 \equiv (1-\rho)\xi(1-\nu)(\nu+\psi)$

$\varpi^c_2 \equiv \rho\omega_1 + (1-\rho)\omega_2 + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$

$\xi \equiv \frac{\eta\sigma}{1-\alpha}$[24]

and where $\tilde{x_t}^{c,i} \equiv \hat{x}^i_t - \hat{x}^{c,i}_t$ and $\hat{x}^{c,i}_t$ indicates the target of the common authority under coordination.

According to (63), under coordination, welfare losses are increasing in inflation, output, private consumption and public expenditure gaps. At the same time, these losses are affected by the gaps of terms of trade[25] and the consequent mis-allocation in private consumption, public expenditure and production which crucially depends on the different degrees of openness and the elasticity of substitution among bundles produced in different country.

The target. The target of the coordinated policymaker is the Pareto efficient and corresponds to the flexible price allocation under technology shocks:

$$\varphi\hat{y}^{c,i}_{H,t} + \gamma\hat{g}^{c,i}_t = (\varphi+1)a^i_t - \frac{\varpi^c_1}{1-\rho}(\hat{c}^{c,i}_t - \hat{c}^{c,*}_t) \tag{64}$$

$$\sigma\hat{c}^{c,i}_t - \gamma\hat{g}^{c,i}_t = \frac{1}{1-\rho}\varpi^c_1(\hat{c}^{c,i}_t - \hat{c}^{c,*}_t) - \frac{1}{\rho}\Big[\varpi^c_1(\hat{g}^{c,i}_t - \hat{g}^{c,*}_t) + \varpi^c_2(\hat{c}^{c,i}_t - \hat{c}^{c,*}_t)\Big] \tag{65}$$

$$\hat{y}^{c,i}_{H,t} + \frac{\psi}{1-\psi}(\hat{y}^{c,i}_{H,t} - \hat{y}^{c,*}_t) =$$
$$\rho\hat{c}^{c,i}_t + (\delta-1)\rho(\hat{c}^{c,i}_t - \hat{c}^{c,*}_t) + (1-\rho)\hat{g}^{c,i}_t - \nu(1-\rho)(\hat{g}^{c,i}_t - \hat{g}^{c,*}_t) \tag{66}$$

[24]See the appendix for the definition of ω_1, ω_2 and ω_3.

[25]Notice in fact that by (35), (36) and (37), $\hat{c_t}^i - \hat{c_t}^*$ is perfectly negative correlated with the terms of trade.

The difference across countries embodied in (64) - (66) are explained as efficient responses to asymmetric shocks to producitvity. If, for instance, its technological shock is above the union average, then a single country economy experiences a terms of trade worsening and efficiently increases the demand for domestic goods relative to those for foreign goods.

However, once the system of equations (64) - (66) is integrated:

$$\varphi\hat{y}_t^{c,*} + \sigma\hat{c}_t^{c,*} = (1+\varphi)a_t^* \quad \gamma\hat{g}_t^{c,*} = \sigma\hat{c}_t^{c,*} \quad \hat{y}_t^{c,*} = \rho\hat{c}_t^{c,*} + (1-\rho)\hat{g}_t^{c,*} \tag{67}$$

Thus, under coordination, the target of the common authority on average corresponds exactly to the target of the policymaker of a closed economy where the only existing distortion is due to price stickiness.

The optimal policy mix. Given (64) - (66), the set of constraints relevant the optimal policy problem - the resource constraint, the Phillips Curve and condition (40) - for can be rewritten in terms of gaps as:

$$\begin{aligned}&\tilde{y}_{H,t}^{c,i} + \frac{\psi}{1-\psi}(\tilde{y}_{H,t}^{c,i} - \tilde{y}_t^{c,*}) = \\ &\rho\tilde{c}_t^{c,i} + (\delta-1)\rho(\tilde{c}_t^{c,i} - \tilde{c}_t^{c,*}) + (1-\rho)\tilde{g}_t^{i,c} - \nu(1-\rho)(\tilde{g}_t^{c,i} - \tilde{g}_t^{c,*})\end{aligned} \tag{68}$$

$$\pi_{H,t}^i = \lambda\left[\varphi\tilde{y}_{H,t}^{c,i} + \sigma\tilde{c}_t^{c,i} + \omega_4((1-\psi)\alpha + \psi)(\tilde{c}_t^{c,i} - \tilde{c}_t^{c,*})\right] + \beta E_t\{\pi_{H,t+1}^i\} + \lambda\hat{\mu}_t^i \tag{69}$$

$$\pi_{H,t}^i - \pi_t^* = -\omega_4(\Delta\tilde{c}_t^{c,i} - \Delta\tilde{c}_t^{c,*}) - \omega_4(\Delta\upsilon_{1,t}^i - \Delta\upsilon_{1,t}^*) \tag{70}$$

for all t and i and where $\upsilon_{1,t}^i \equiv \hat{c}_t^{c,i}$.

This system of equations makes clear which are the tradeoffs of the common policymaker under coordination. From the union wide perspective, there is only tradeoff due to the presence of markup shocks. As in a closed economy, a markup shock affects inefficiently firms marginal costs making incompatible to fully stabilize both inflation and output gap. Nevertheless, when shocks are just to technology, the optimal policy mix - the efficient provision of the public goods and (average) strict inflation targeting-allows to close all the gaps and reach on average the efficient allocation.

This is possible only at the average union level. At the single country level, independently of which type of shocks hits the economy, the adoption a common currency implies always a tradeoff between inflation and output stabilization: if the nominal exchange rates are fix and prices are sticky, the terms of trade cannot adjust instantaneously in response to asymmetric shocks and the flexible prices allocation (in the case the first best allocation) cannot be implemented. Therefore, as long as shocks are asymmetric:

$$\pi_{H,t}^i \neq 0 \tag{71}$$

for all i for some t. This explains why, as highlighted by Galí and Monacelli (2008), under coordination there is room to use the single country government expenditure as a stabilization tool.

5 The case of no-coordination

Under no-coordination, fiscal authorities are coordinated neither among each other nor with the common central bank. The monetary and fiscal policy problems are then formulated as follows[26].

Single countries' governments maximize the welfare of the small open economy representative agent:

$$\sum_{t=0}^{\infty}\beta^t E_0\left[\frac{C_t^{1-\sigma}}{1-\sigma}+\chi\frac{G_t^{1-\gamma}}{1-\gamma}-\frac{1}{\varphi+1}\left(\frac{Y_{H,t}}{A_t}\right)^{\varphi+1}\right] \tag{72}$$

with respect to C_t, Y_t, $Y_{H,t}$, $\Pi_{H,t}$, Z_t, F_t and K_t, subject to the single country equilibrium conditions (39), (44), (45), (52), (53), (55), and (56), where $P_t/P_{C,t}$, $P_{G,t}/P_{C,t}$, $P_{H,t}/P_t$ are determined according to (35), (36), (37) and taking as given the union average variables including the nominal interest rate chosen by the common central bank.

Conversely, the monetary authority maximizes the average union welfare:

$$\sum_{t=0}^{\infty}\beta^t E_0\int_0^1\left[\frac{C_t^{i\,1-\sigma}}{1-\sigma}+\chi\frac{G_t^{i\,1-\gamma}}{1-\gamma}-\frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1}\right]di \tag{73}$$

with respect to C_t^i, Y_t^i, $Y_{H,t}^i$, $\Pi_{H,t}^i$, Z_t^i, F_t^i and K_t^i for all i, subject to equilibrium conditions (39), (44), (45), (52), (53), (55), and (56) for all i, where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$ are determined according to (35), (36), (37) and taking as given fiscal policies[27].

Given the formulation of the monetary and fiscal policy problems, it can be shown that at the symmetric deterministic steady state, zero inflation is a Nash equilibrium policy[28]. In particular, the optimality conditions evaluated at the zero inflation steady state lead to:

$$C^{-\sigma}=(1-\psi)\left[\delta_1+\delta_2\frac{G}{C}+\delta_3\frac{Y}{C}\right]Y^{\varphi} \tag{74}$$

$$\chi G^{-\gamma}=(1-\psi)(1-\nu)Y^{\varphi} \tag{75}$$

$$Y=C+G \tag{76}$$

where $A=1$. The comparison between the systems of equations (74) - (76) and (60) - (62) makes clear that, when uncoordinated, fiscal policymakers generate static distortions. Indeed at the symmetric steady state, under coordination, the MRS between both leisure and private consumption and leisure and public expenditure are set equal to the correspondent MRT; instead under no-coordination they are respectively determined by $(1-\psi)\left[\delta_1+\delta_2\frac{G}{C}+\delta_3\frac{Y}{C}\right]>1$ and $(1-\psi)(1-\nu)<1$. In other words,

[26] For more details, see the appendix.

[27] Namely G_t^i and τ^i for all countries and in all periods.

[28] See the appendix.

uncoordinated fiscal authorities have an incentive to expand the government spending and to tax labour beyond the efficient level. The interpretation of this findings is the following. By boosting the demand for domestic goods and reducing its supply, autonomous governments seek to improve their terms of trade. They realize that rendering home produced goods more expensive than foreign goods raises profits revenue of households and makes up for the reduction in labor income and the increase in lump-sum taxes. Then, households can consume more public goods and work less than under coordination. The decrease in private consumption due to the terms of trade improvement is more than compensated by the higher public good provision and the lower labor effort. In this way countries seek to externalize both the costs of production and taxation to foreign cosumers[29]. Obviously given that at the symmetric steady state everybody is doing the same, in equilibrium the prices of all goods are equal and everybody is worse off.

To get a sense of the magnitude of the inefficiency generated at the steady state by uncoordinated policies it is sufficient to look at the following table:

	Coordination	No-Coordination
$\frac{C}{Y}$	0.97	0.73

Under the baseline calibration, according to which $\alpha = 0.4$, $\nu = 0.2$ and $\psi = 0.4$[30], if fiscal policies are not coordinated the steady state consumption output ratio is equal to the 73% (as in the European Monetary Union) whereas, if they are coordinated, it reaches 97%. In other words at the steady state governments' size is highly inefficient. And as it will be clear in the next subsections this static distortion will be key even in determining the effects that that lack of coordination produces over the business cycle.

5.1 Fiscal policy

The welfare approximation. For the fiscal policymaker the single country welfare has been approximated as:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\frac{\varepsilon}{\lambda}(\pi_{H,t})^2 + \varphi(\tilde{y}^f_{H,t})^2 + \varpi^f_1(\tilde{g}_t{}^f)^2 + \varpi^f_2(\tilde{c}_t{}^f)^2 + 2\varpi^f_3\tilde{g}_t{}^f\tilde{c}_t{}^f\Big] \\ +t.i.f.p. \tag{77}$$

with

$$\varpi^f_1 \equiv (1-\rho)(1-\psi)(1-\nu)\gamma$$

$$\varpi^f_2 \equiv \rho(1-\psi)\delta\sigma + \varsigma_2$$

$$\varpi^f_3 \equiv (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi)$$

where ς_1 and ς_2 are properly defined in the appendix, $t.i.f.p$ stands for terms independent of fiscal policy, $\tilde{x}^f_t \equiv \hat{x}_t - \hat{x}^f_t$ and $\hat{x}^f_t$ indicates the target of the fiscal authority.

[29]In fact at the steady state the incentive to over-expand the government expenditure is present even when the labor supply is completely inelastic.

[30]and that will be discussed in details below.

Variables, target and weights that enter in this loss are different than those under coordination. Fiscal authorities take as given the average union allocation and weight more fluctuations in the private consumption gap and less those in the public expenditure gap.

The target. The target of the fiscal policymaker is determined by the following conditions:

$$\varphi \hat{y}_{H,t}^{f} + \gamma \hat{g}_{t}^{f} = (\varphi + 1)a_t - ((1-\rho)(1-\psi)(1-\nu))^{-1}\,\varpi_3^{f}(\hat{c}_t^{f} - \hat{c}_t^{*}) \tag{78}$$

$$\begin{aligned} &\varphi \hat{y}_{H,t}^{f} + \sigma \hat{c}_{t}^{f} = (\varphi + 1)a_t - ((1-\psi)\rho\delta)^{-1}\Big[\varpi_3^{f}(\hat{g}_t^{f} - \hat{g}_t^{*}) + \varsigma_2(\hat{c}_t^{f} - \hat{c}_t^{*}) \\ &+(1-\psi)\delta_3(\hat{y}_t^{*} - \hat{c}_t^{*}) + (1-\psi)(1-\rho)\delta_2(\hat{g}_t^{*} - \hat{c}_t^{*})\Big] \end{aligned} \tag{79}$$

$$\begin{aligned} &\hat{y}_{H,t}^{f} + \frac{\psi}{1-\psi}(\hat{y}_{H,t}^{f} - \hat{y}_t^{*}) = \\ &\rho \hat{c}_t^{f} + (\delta - 1)\rho(\hat{c}_t^{f} - \hat{c}_t^{*}) + (1-\rho)\hat{g}_t^{f} - \nu(1-\rho)(\hat{g}_t^{f} - \hat{g}_t^{*}) \end{aligned} \tag{80}$$

That target is the efficient allocation from the small open economy point of view[31]. It corresponds to the flexible price allocation in the hypotheses that independent governments are not coordinated. To grasp some insights on the incentives of these authorities and the dynamic union wide effects produced by their policy choices, assume that shocks are symmetric. Then if *implemented* the target determined by (78), (79) and (80) satisfies the following conditions:

$$\varphi \hat{y}_t^{f} + \gamma \hat{g}_t^{f} = (1+\varphi)a_t^{*} \tag{81}$$

$$\delta\rho(\gamma \hat{g}_t^{f} - \sigma \hat{c}_t^{f}) = \delta_3(\hat{y}_t^{f} - \hat{c}_t^{f}) + (1-\rho)\delta_2(\hat{g}_t^{f} - \hat{c}_t^{f}) \tag{82}$$

$$\hat{y}_t^{f} = \rho \hat{c}_t^{f} + (1-\rho)\hat{g}_t^{f} \tag{83}$$

This set of conditions (that is nothing more than the log linear approximation of (74) - (76)) differs from the target of the coordinated authority in two main respects.

First, as already pointed out under no-coordination the government expenditure share in output is inefficiently high because at the steady state both government expenditure is over-expanded and output under-produced. Under the baseline calibration, this static distortion implies a clear consequence: it inefficiently amplifies the impact of the government expenditure shocks over output fluctuations. Indeed, according to the baseline calibration, the intertemporal elasticity of substitution of the government expenditure, γ^{-1}, is higher than that of private consumption, σ^{-1}. Hence over the cycles policymakers want to substitute private consumption with public expenditure in order to smooth the path of the more inelastic bundle. As a consequence public expenditure fluctuates more than private consumption. Then, being steady state public expenditure share in output inefficiently high, one percent increase in government spending would expand more output under no-coordination than under coordination.

[31]It is recovered by maximizing the purely quadretic approximation of (72) recovered in the appendix, subject to condition (32) and taking as given the average union variables

Secondly according to (82), $\widehat{mrs}_t$ between private consumption and public expenditure is not equal on average to the corresponding $\widehat{mrt}_t$ as it would be under coordination. This is because uncoordinated fiscal authorities try to influence the terms of trade to their own advantage even over the business cycle. Indeed, as long as $\gamma \neq \sigma$ and intermediate or public goods are traded [32], they have an incentive to generate procyclical response of the average public spending beyond the efficient provision of the public goods. In other words they seek to inefficiently boost the volatility of public consumption and dampen that of labour in order to reduce the volatility of the terms of trade, output and private consumption. The underlying reason that explains this behaviour is the attempt to reduce the cost of uncertainty for domestic consumers - that are risk adverse - by externalizing business cycle fluctuations to other countries' households.

The optimal policy. Fiscal policymakers maximize (77) subject to (47), (57) and (40) properly rewritten in terms of gaps:

$$\tilde{y}^f_{H,t} = (1-\psi)\delta\rho\tilde{c}_t{}^f + (1-\psi)(1-\nu)(1-\rho)\tilde{g}^f_t \tag{84}$$

$$\pi_{H,t} = \lambda\left[\varphi\tilde{y}^f_{H,t} + \omega_4\tilde{c}^f_t\right] + \beta E_t\{\pi_{H,t+1}\} + \lambda(\hat{\mu}_t + \upsilon_{2,t}) \tag{85}$$

$$\pi_{H,t} = -\omega_4\Delta\tilde{c}^f_t + \upsilon_{3,t} \tag{86}$$

where

$\upsilon_{2,t} \equiv \varpi^f_4(\hat{c}^f_t - \hat{c}^*_t) - ((1-\psi)\rho\delta)^{-1}\Big[\varpi^f_3(\hat{g}^f_t - \hat{g}^*_t) + (1-\psi)\delta_3(\hat{y}^*_t - \hat{c}^*_t) + (1-\psi)(1-\rho)\delta_2(\hat{g}^*_t - \hat{c}^*_t),$

$\upsilon_{3,t} \equiv \pi^*_t - \omega_4(\Delta\hat{c}^f_t - \Delta\hat{c}^*_t)$

$\varpi^f_4 \equiv (\omega_4((1-\psi)\alpha + \psi) - ((1-\psi)\rho\delta)^{-1}\varsigma_2).$

According to first order conditions of this problem with respect to $\tilde{y}^f_{H,t}$, $\tilde{g}^f_t$, $\tilde{c}^f_t$ and $\pi_{H,t}$:

$$\begin{aligned}\pi_{H,t} = &-\frac{1}{\varphi\varepsilon}A(L)\left[\varphi\tilde{y}^f_t + \gamma\tilde{g}^f_t + ((1-\rho)(1-\psi)(1-\nu))^{-1}\varpi^f_3\tilde{c}^f_t\right] \\ &+\frac{\lambda}{\omega_4\varepsilon}B(L)\left[(1-\psi)\delta\rho(\varphi\tilde{y}^f_t + \sigma\tilde{c}^f_t) + \varpi^f_3\tilde{g}^f_t + \varsigma_2\tilde{c}^f_t\right]\end{aligned} \tag{87}$$

where $A(L) \equiv \left[(1-L) + \lambda\left[\frac{(1-\psi)\delta\rho\varphi}{\omega_4} + 1\right]B(L)\right]$ and $B(L) \equiv (1 - E_tL^{-1})^{-1}$.

This system of equations (84)-(87) determines the gaps of the small open economy under uncoordinated fiscal policies for a given path of the aggregate variables [33]. According to these conditions in general, uncoordinated governments always face

[32] i.e. even $\psi > 0$ or $\nu > 0$ Note that if $\psi > 0$ or $\nu > 0$ then $\delta_3 > 0$ or $\delta_2 > 0$.

[33] To recover the average union allocation one has to find the optimal average level of provision of public expenditure and then determines the average union private consumption and output using the other equilibrium conditions: the average union resource constraint, the average union Phillips curve and the IS curve.

a tradeoff between stabilizing inflation and output gap. Indeed, differently from what happens under coordination they are not able to achieve their target even in the absence of idiosyncratic and markup shocks. This is made made clear by condition (85): unless special parametric restrictions are met, it is not possible to fully stabilize the $\widehat{mrs}_t$ between private consumption and leisure at the desired level despite the complete stabilization of the home inflation. The key reason of this outcome is the incentive of independent fiscal authorities to affect the terms of trade in their favour even over the business cycles. As already emphasized, these policymakers, in fact, want the $\widehat{mrs}_t$ between private consumption and leisure to fluctuate less than its $\widehat{mrt}_t$ to restrain private consumption and output volatility at foreign consumers' expense.

However there are specific restrictions under which this result may be reversed and the optimal fiscal policy is consistent with home price stability. Specifically if the intermediate and public goods are not traded - i .e. $\psi = 0$ and $\nu = 0$ - or the intertemporal elasticities of substitution of public and private consumption are equal - $\gamma = \sigma$ - $\pi_{H,t} = 0$ for all t is optimal as long shocks are symmetric and to technology. To see why consider: 1) If shocks are symmetric there is no additional trade off generated by the adoption of a common currency (i.e. $\upsilon_{3,t} = 0$) 2) If $\psi = 0$ and $\nu = 0$ or $\gamma = \sigma$ then stabilizing the $\widehat{mrs}_t$ between both private and public consumption and private at the $\widehat{mrt}_t$ is the target of the uncoordinated fiscal authorities (then i.e. $\upsilon_{2,t} = 0$). And in fact it can be shown that under these restrictions all the conditions (84)-(87) are simultaneously satisfied.

Nevertheless notice that this last finding is conditional on the willingness of the monetary policymaker to completely stabilize inflation. Whether she finds it optimal or not it will be clarified in the next paragraph.

5.2 Monetary policy

The welfare approximation. Under fiscal policy no-coordination the objective of the common central bank can be approximated as:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\left[\zeta_3\frac{\varepsilon}{\lambda}(\pi_t^*)^2 + \zeta_3\varphi(\tilde{y}_t^{m,*})^2 + \varpi_1^m(\tilde{c}_t^{m,*})^2 + 2\zeta_2\sigma\tilde{c}_t^{m,*}\tilde{y}_t^{m,*}\right]$$
$$+s.o.t.i.m.p. \tag{88}$$

with

$\varpi_1^m \equiv \rho\delta(\sigma-1) + \zeta_1\rho - \zeta_2\sigma^2$

$\zeta_1 \equiv \frac{(1-\psi)\delta\varphi\rho+\sigma}{\varphi\rho+\sigma}$

$\zeta_2 \equiv \frac{((1-\psi)\delta-1)\rho}{\varphi\rho+\sigma}$

$\zeta_3 \equiv \frac{(\varphi+1)(1-\psi)\delta\rho+\sigma-\rho}{\varphi\rho+\sigma}$

where $\tilde{x}_t^{m,i} \equiv \hat{x}_t^i - \hat{x}_t^{m,i}$ and $\hat{x}_t^{m,i}$ is the target for $\hat{x}_t^i$ chosen by the central bank.

The objective approximation of the central bank under no-coordination diverges from those of the uncoordinated fiscal authority and of the common policymaker under coordination. And this not only because the central bank does not choose the optimal provision of the public goods. Indeed even abstracting from this consideration, there are striking differences in target, weights and variables that enter in the approximation. The key determinant of these divergences is the steady state distortion as shown by the dependence of the weights and of the average target from ρ, ζ_1, ζ_2 and ζ_3.

The target. The target of the central bank can be retrieved from[34]:

$$\varphi\hat{y}_t^{m,*} + \sigma\hat{c}_t^{m,*} = (1+\varphi)a_t^* - \frac{\zeta_2}{\zeta_3}\left[\frac{\sigma}{\rho}(\hat{y}_t^{m,*} - \hat{c}_t^{m,*}) + (1+\varphi)\hat{\mu}_t^*\right] \tag{89}$$

$$\hat{y}_t^{m,*} = \rho\hat{c}_t^{m,*} + (1-\rho)\hat{g}_t^* \tag{90}$$

According to (89) and (90) in general, the target of common central bank is not the first best and therefore does not coincide with the target of the coordinated common authority under either technological or markup shocks. This is due to various reasons.

Some of these reasons have been already underlined in the previous paragraph. First, under the baseline calibration fiscal shocks expand sub-optimally output fluctuations because at the steady state the governments' size is inefficiently high; secondly, autonomous governments try to manipulate the terms of trade even over the business cycles. Hence, in general uncoordinated fiscal policies produce average dynamic distortions that need to be internalised in the target and in the policy decisions of the common central bank. It is worth to notice that, as made explicit by (89) and (90), the impact of these spillovers on the monetary policy choices depends crucially on the difference between $\hat{y}_t^{m,*}$ and $\hat{c}_t^{m,*}$. In fact, the closer are $\hat{y}_t^{m,*}$ and $\hat{c}_t^{m,*}$, the less is the intratemporal substitution between private consumption and output - and then the business cycle distortions generated by this substitution -, the closer is target of the common central to the average flexible price allocation. Actually when $\hat{y}_t^{m,*} = \hat{c}_t^{m,*}$ the effects of these dynamic inefficiencies disappear on average.

The other reason that explains the influence of independent governments' decisions on the target of the common central bank is related to the state distortion and the long run effects of monetary policy. According to (89) and differently from the case of coordination the target of the common central bank does react to markup shocks. Why? To answer this question, consider the special case in which shocks are symmetric and just to the markup. Then, under flexible prices, from condition (13), it follows:

$$E\left\{\frac{W_t}{P_{C,t}}\right\} = E\left\{\Gamma(Y_t)\right\} E\left\{(1+\mu_t)\right\} + Cov\left\{(1+\mu_t)\Gamma(Y_t)\right\} \tag{91}$$

where $\Gamma(Y_t) \equiv Y_t^{\varphi}(Y_t - G_t)^{\sigma}$ and $\Gamma_Y(Y_t) > 0$.

According to (91), the lower is the covariance between markup shocks and output, the higher is the average per-period output for a given level of the average per-period real wage. In other words, if is an increase in output fluctuations in response to markup shocks - which corresponds to a decrease of the covariance between the markup and

[34]This target is determined by maximizing the purely quadratic approximation of (73) shown in the appendix subject to (32) and taking as given $\hat{g}_t^*$.

output given that a positive markup shock tends to reduce output - domestic consumers have to increase their average labour effort if they want to maintain the same average real wage. As a consequence, the common central bank recognizes that monetary policy can have beneficial effects in the long run: by becoming more aggressive in fighting inflation and allowing for an increase in output fluctuations it can shift upward the average labour supply curve generating an efficient increase in the long run average level of output.

The constraints to the monetary policy problem can be rewritten in terms of gaps as:

$$\tilde{y}_t^{m,*} = \rho \tilde{c}_t^{m,*} \tag{92}$$

$$\pi_t^* = \lambda \left[\varphi \tilde{y}_t^{m,*} + \sigma \tilde{c}_t^{m,*} \right] + \beta E_t\{\pi_{t+1}^*\} + \lambda(\frac{1}{\zeta_3}\hat{\mu}_t^* + \upsilon_{5,t}^*) \tag{93}$$

with
$\upsilon_{5,t}^* \equiv -\frac{\zeta_2}{\zeta_3}\frac{\sigma}{\rho}(\hat{y}_t^{m,*} - \hat{c}_t^{m,*})$

Moreover, the system of optimality conditions of the monetary policymaker can be rewritten as:

$$\pi_t^* = -\frac{\rho(1-L)}{\varepsilon(\sigma+\varphi\rho)} \left[\varphi \tilde{y}_t^{m,*} + \sigma \tilde{c}_t^{m,*} + \frac{\zeta_2}{\zeta_3}\frac{\sigma}{\rho}(\tilde{y}_t^{m,*} - \tilde{c}_t^{m,*}) \right] \tag{94}$$

Thanks to this set of equations it is possible to recover the average union allocation determined by the optimal reaction of the common central bank to given fiscal policies. Clearly given the changes in the target and as stressed by (93), the common central bank faces different tradeoffs than those of the policy authority under coordination. On the one hand, in general, even if shocks are only to technology, strict inflation targeting is not optimal ($\upsilon_{5,t}^* \neq 0$). A result this that contrasts with the findings of Galí and Monacelli (2008) and Beetsma and Jensen (2005). Fully stabilizing the average union inflation is not optimal because it does not allow to close the average output gap. Indeed the flexible price allocation is not efficient and the monetary authority wants to seek to correct the dynamic distortions due to the lack of coordination among fiscal policymakers. On the the other hand if shocks are to the markup, the central bank tries to stabilize more inflation than output with respect to the case of coordination. It realizes that an increase in output fluctuations in response to markup shocks can boost in the long run the inefficiently low level of output.

Now it is possible to the question posed at the end of last the paragraph. In presence of productivity shocks under which conditions does the central bank find it optimal to completely stabilize the average union inflation?

Suppose that according to a policy rule $\hat{g}_t^* = \hat{c}_t^{m,*}$ for all t and there are only technological shocks. In that case $\pi_t^* = 0$ for all t satisfies conditions (92), (93) and (94). However when $\hat{g}_t^* \neq \hat{c}_t^{m,*}$ for some t, then $\pi_t^* = 0$ for all t cannot be optimal. Thus even when there is no trade in public and intermediate goods, namely $\nu = \psi = 0$ the monetary policymaker would not stabilize the average union inflation even under symmetric productivity shocks, while in that case fiscal authorities would be willing to do that.

5.3 The case for average price stability

The analysis of the previous sections allows to formulate the following proposition:

Proposition 1 *Under fiscal policy no-coordination, $\pi^*_t = 0$ for all t is an Nash equilibrium outcome of the monetary and fiscal policy game if and only if $\sigma = \gamma$ and shocks are to technology.*

Proof.1 See the appendix. ■

Proposition 1 can be interpreted as follows: when $\sigma = \gamma$ and shocks are to technology, the lack of coordination among fiscal policymakers does not yields on *average* only *static* distortions[35] namely the steady state distortions. Indeed under this parametric restriction and the average union inflation is completely stabilized, two conditions are simultaneously satisfied. On the one hand the average marginal rates of substitution between private and public consumption and private consumption and leisure fluctuate as the marginal rates of transformation between the same variables, i.e. $\sigma\hat{c}^*_t - \gamma\hat{g}^*_t = 0$ and $\varphi(\hat{y}^*_t - \hat{a}^*_t) + \sigma\hat{c}^*_t = \hat{a}^*_t$; on the other hand the average union output co-move with the private and public consumption, i.e. $\hat{y}^*_t = \hat{g}^*_t = \hat{c}^*_t$. These two conditions ensure that, even if fiscal policies are uncoordinated, under flexible prices, the average union fluctuations of output, and public and private consumption replicate the fluctuations that would be achieved if the fiscal policies were coordinated. As a consequence the monetary authority seeks to remove the only remaining distortion that can be corrected: the average price stickiness. In fact stabilizing completely the average inflation is optimal: it allows at the same time to eliminate on average the inefficiencies produced by price rigidities and to keep the average allocation at the constrained-efficient level.

5.4 The general case

This section analyzes the general case allowing for different intertemporal elasticities of substitution of private and public consumption and different kinds of shocks. These differences generates an incentive for the fiscal authorities to seek to substitute intratemporally the public and private consumption. In the case of different elasticities in order to smooth intertemporally the path of more inelastic goods. In the case of markup shocks in order to reduce the home country private consumption and output gap. As a result because of this intratemporal substitution, it is no more true that, under technological shocks, the symmetric allocation is proportional to the efficient one. And both monetary and fiscal policies at the average union level do not correspond to the ones that are optimal under coordination. In fact neither under technological shocks the common central bank should seek to pursue price stability nor fiscal policies ensure on average the efficient provision of public goods.

[35]...at least up to a first order approximation of the optimal policies.

5.5 Calibration

Impulse responses to a one percent rise in technology and markup under optimal policies are recovered using the calibration indicated in the appendix which is close to those of Galí and Monacelli (2008) and Galí and Monacelli (2005). In particular γ^{-1} and φ^{-1} the intertemporal elasticities of substitution of public consumption and labor, α the degree of openness in private consumption, ε, the elasticity of substitution among goods produced in the same country, β the preferences discount factor, θ the parameter that governs the level of price stickiness in the economy and ac the first order autocorrelation of shocks[36] are set according to their calibration. Conversely σ^{-1} the intertemporal elasticity of substitution of private consumption and η the elasticity of substitution between bundles produced in different countries is set according to Benigno and Benigno (2006), ψ the degree of openness in the intermediate goods is equal to α and $\nu = 0.2$ as partially suggested by Brülhart and Trionfetti (2004). Finally χ the parameter that regulates the relative weight of the public good in the preferences is calibrated to match the average consumption output ratio of European Monetary Union.

5.6 Dynamic Simulations

The appendix shows the impulse responses to a one percent increase in aggregate technology and markup under the optimal policies. They may be interpreted as follows.

Technological shocks. When shocks are to technology and fiscal policies are coordinated, the optimal policy mix embodies two clear prescriptions for the average union economy: the nominal interest rate should be set to fully stabilize the average inflation, while the government expenditure should ensure, on average, the efficient provision of the public goods. These policies allow to close all the gaps at the union level and reach the efficient fluctuations. However under fiscal policy no-coordination none of these prescriptions is still valid. The first is not valid because of the dynamic effects produced by uncoordinated fiscal policies. In particular, given that $\gamma < \sigma$ and the incentive of independent fiscal authorities to manipulate the terms of trade in their favor, a technology shock increases more the provision of the public goods than the private consumption. As a consequence, because of the inefficiently high share of government expenditure in output, there is an overexpansion of output. Thus, the common central bank has to trade off between stabilizing the average union inflation and reducing the output gap. This explains why the monetary policy allows for a certain degree of average union deflation, being more restrictive under no-coordination that under coordination (as emphasized by the different path of the nominal interest rates). Obviously in these circumstances not even the average public good provision is efficient. Fiscal policymakers seek to implement a beggar-my-neighbour policy even over the business cycle, disregarding the aggregate distortions resulting from their joint action. By overexpanding the provision of the public good beyond the efficient level, they want to reduce the terms of trade volatility in order to externalize the cost of private consumption and output fluctuations to other country consumers. And in fact according to the impulse responses the government expenditure expansion is greater than σ/γ that of

[36] Both markup and productivity shocks are suppose to be AR(1).

private consumption.

Markup shocks. When shocks are to the markup, the policy prescriptions under coordination are twofold. Fiscal policy is not a useful tool to stabilize the average effects of the markup shocks: for this purpose it is more efficient to use the nominal interest rate which is a costless instrument. Therefore under markup shocks, the average union government expenditure should be kept at the steady state level. At the same time, the monetary authority should trade off between stabilizing inflation and closing the output gap given the consequences of an inefficient shock to the mark up. The policy prescriptions under no-coordination are quite different. First, because in response to a positive markup shock the optimal monetary policy becomes more aggressive in reducing inflation than under coordination. Indeeed as made clear by (91), the common central bank wants output to fluctuate more in response to markup shocks because in this way it induces domestic consumers to augment their per period labour supply. As a result there is a beneficial increase in the inefficiently low level of the per period output. Indeed according to the impulse responses, the nominal interest rate is higher, while the average inflation and output are lower, under no-coordination than under coordination. Secondly, because autonomous governments lower the provision of public goods. This is the result of the balance between different objectives. On the one hand the aggregate markup shock induce a fall in the average union private consumption and output contracting the foreigners' demands for home produced goods. In response to these external shocks perceived as efficient, the non-coordinated policymaker would like to decrease domestic private and public consumption increasing the leisure[37]. However she has to trade off between this purpose and stabilizing the undesired effect of the domestic markup shock: the boost in the home inflation and output gap. Thus, the provision of public goods falls, but not much more than the private consumption in order to alleviate the reduction of the private consumption itself that actually after the first periods is higher than under coordination. Thus, while under coordination, the common authority recognizes that only the monetary policy should be used to stabilize the average effects of markup shocks, under no-coordination the single country government takes as given the actions of the other policymakers and tries on its own to stabilize the effects of the domestic markup shock in its country.

6 Conclusions

According to this paper within a monetary union the lack of coordination among fiscal policymakers has relevant implications for both optimal monetary and fiscal polcies. In fact, only under a special parametric restriction and when shocks are to technology, fiscal policy no-coordination does not matter for the optimal monetary policy design. However, in general, this result is not verified and as opposed to the case of coordination under no-coordination it is possible to reach the following conclusions: first when shocks are to technology, stabilizing the average union prices is not optimal; second under markup shocks, the monetary authority is mainly focused on the stabilization of the average union inflation. Finally even if shocks are symmetric, fiscal policies are used

[37]This is made clear by the (78)-(80).

as stabilization tool.

The analysis of the interactions between monetary and fiscal policies with a monetary union - for the cases of coordination and no-coordination- may be extended in several directions. On the one hand there is scope for relaxing some of the key assumptions of the model used in this paper, for instance, by introducing sticky wages and allowing for incomplete financial markets. On the other hand future research should investigate the implications both at the steady state and over the business cycles of different monetary and fiscal policy games, as the Stackelberg one in which the monetary authority chooses its policy before the uncoordinated fiscal policymakers.

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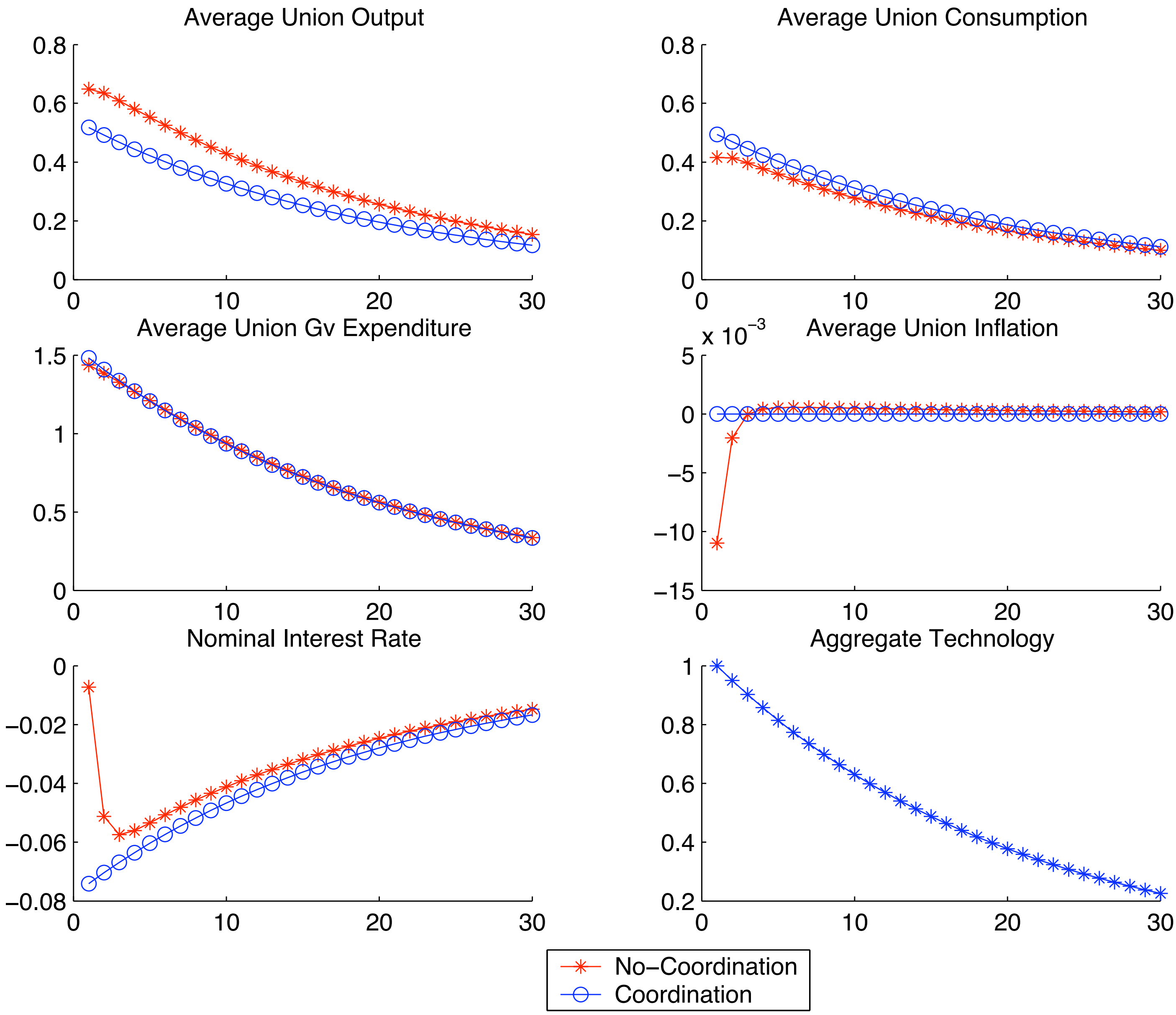
APPENDIX

Baseline Calibration

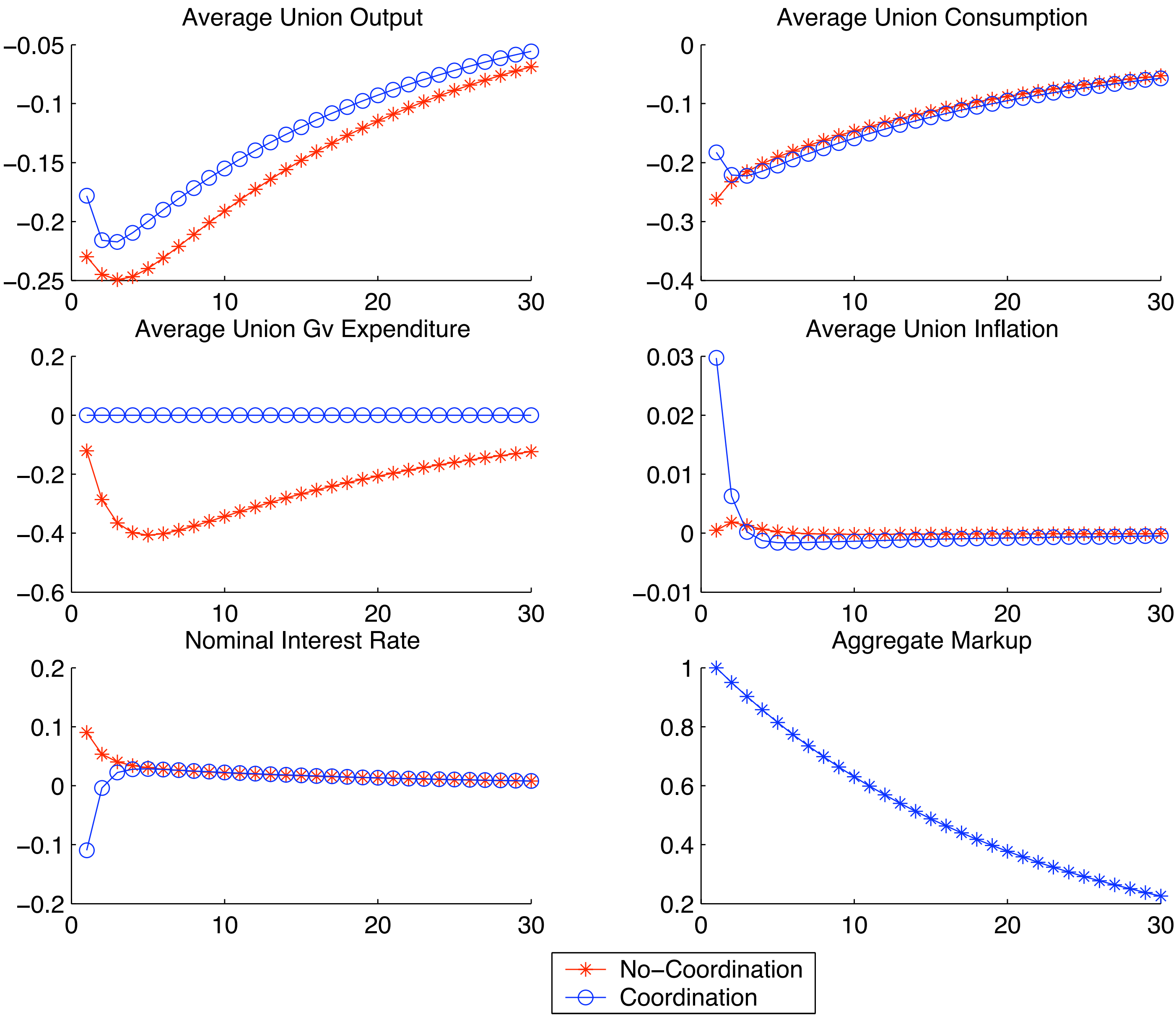
$\sigma^{-1} = 1/3$	Intertemporal elasticity of substitution of the private goods;
$\gamma^{-1} = 1$	Intertemporal elasticity of substitution of the public goods;
$\eta = 4.5$	Elasticity of substitution between home and foreign private goods;
$\varphi^{-1} = 1$	Intertemporal elasticity of substitution of labor;
$1 - \alpha = 0.6$	Degree of home bias in the private bundle;
$1 - \nu = 0.8$	Degree of home bias in the public bundle;
$1 - \psi = 0.6$	Degree of home bias in the intermediate input;
$\chi = 0.03$	Weight of the public bundle in the preferences.
	It implies a steady state consumption output ratio under no-coordination of 0.73;
$\varepsilon = 6$	Elasticity of substitution among goods produced in the same country;
$\beta = 0.99$	Preferences discount factor;
$ac = 0.95$	Autocorrelation of shocks;

Impulse responses to a one percent rise in the technology



Variables are expressed as log-deviations from the steady state

Impulse responses to a one percent rise in the markup



Variables are expressed as log-deviations from the steady state

Proof of proposition 1

First part of the proof. If $\gamma = \sigma$ for all t then it can be shown that $\tilde{g}_t^{f,*} = \tilde{c}_t^{f,*} = \tilde{y}_t^{f,*}$ $\pi_t^* = 0$ satisfies the average of conditions (87)-(86). Then $\hat{g}_t^* = \hat{c}_t^* = \hat{y}_t^*$ which implies that $\tilde{g}_t^{m,*} = \tilde{c}_t^{m,*} = \tilde{y}_t^{m,*} = 0$.

The second part of the proof can be obtained by contradiction. If $\pi_t^* = 0$ for all t, then by (94) and (93) $\hat{c}_t^{m,*} = \hat{y}_t^{m,*}$ which implies that $\hat{c}_t^* = \hat{g}_t^*$. However $\hat{c}_t^* = \hat{g}_t^*$ is consistent with the average of conditions (87)-(86) only if only if $\gamma = \sigma$ which contradicts our initial hypothesis.

The zero inflation deterministic steady states

The policy problem under coordination

Under coordination, the policy maker maximizes the following lagragian with respect to C_t^i, G_t^i, Y_t^i, $Y_{H,t}^i$, Z_t^i, K_t^i, F_t^i and $\Pi_{H,t}^i$ for all i and t:

$$
\begin{aligned}
L = \sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \Bigg\{ & \frac{{C_t^i}^{1-\sigma}}{1-\sigma} + \chi \frac{{G_t^i}^{1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i Z_t^i}{A_t^i}\right)^{\varphi+1} \\
& + \lambda_{1,t}^{i,c}\left[Y_t^i - \left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta}\left((1-\alpha)C_t^i + \alpha {C_t^i}^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}^i}{P_{G,t}^i}\right)^{-\eta} G_t^i + \nu {C_t^i}^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right)\right] \\
& + \lambda_{2,t}^{i,c}\left[Y_{H,t}^i - \left(\frac{P_{H,t}^i}{P_t^i}\right)^{-\eta}\left((1-\psi)Y_t^i + \psi\left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta} {C_t^i}^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right)\right] \\
& + \lambda_{3,t}^{i,c}\left[K_t^i - \left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1} {Z_t^i}^{\varphi}(1-\tau)(1+\mu_t^i)\frac{\varepsilon}{\varepsilon-1}\right] - \lambda_{3,t-1}^{i,c}\theta {\Pi_{H,t}^i}^{\varepsilon} K_t^i \\
& + \lambda_{4,t}^{i,c}\left[F_t^i - Y_{H,t}^i {C_t^i}^{-\sigma}\frac{P_t^i}{P_{C,t}^i}\frac{P_{H,t}^i}{P_t^i}\right] - \lambda_{4,t-1}^{i,c}\theta {\Pi_{H,t}^i}^{(\varepsilon-1)} F_t^i \\
& + \lambda_{5,t}^{i,c}\left[F_t^i - K_t^i\left(\frac{1-\theta {\Pi_{H,t}^i}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right] \\
& + \lambda_{6,t}^{i,c}\left[Z_t^i - \theta Z_{t-1}^i {\Pi^i}_{H,t}^{\varepsilon} - (1-\theta)\left(\frac{1-\theta {\Pi_{H,t}^i}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] \\
& + \lambda_{7,t}^{i,c}\left[\left(\frac{C_t^i}{C_{t-1}^i}\right)^{-\sigma}\left(\frac{C_{t-1}^*}{C_t^*}\right)^{-\sigma}\Pi_t^* - \frac{P_{C,t}^i}{P_t^i}\frac{P_t^i}{P_{H,t}^i}\Pi_{H,t}^i \frac{P_{H,t-1}^i}{P_{t-1}^i}\frac{P_{t-1}^i}{P_{C,t-1}^i}\right]\Bigg\} di
\end{aligned}
$$

where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$, C_t^*, $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$ are determined according (35), (36), (37), (33) and (46) and $Z_{-1} = 1$According to the first order conditions evaluated at the zero inflation symmetric non-stochastic steady state:

$$C^{-\sigma} = \lambda_1^c - \lambda_4^c \sigma Y C^{-\sigma-1}$$
$$\chi G^{-\gamma} = \lambda_1^c$$
$$\lambda_1^c = \lambda_2^c$$
$$Y^{\varphi} = \lambda_2^c - \lambda_3^c(\varphi+1)Y^{\varphi}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} - \lambda_4^c C^{-\sigma}$$
$$Y^{\varphi+1} = -\lambda_3^c \varphi Y^{\varphi+1} + \lambda_6^c(1-\theta)$$
$$\lambda_3^c(1-\theta) = \lambda_5^c$$
$$\lambda_4^c(1-\theta) = -\lambda_5^c$$
$$\lambda_3^c \theta \varepsilon K = -\lambda_4^c \theta(\varepsilon-1)F + \lambda_5^c \frac{\theta}{1-\theta}K$$

If $(1-\tau) = (1/(1+\mu))(\varepsilon-1)/\varepsilon$[38], this system of equations jointly with (38), (44), (45), (52), (53), (55), and (56) can be satisfied by the following solution:

$$C^{-\sigma} = Y^{\varphi}$$
$$\chi G^{-\gamma} = Y^{\varphi}$$
$$Y = C + G$$
$$F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}}{1-\theta}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1}$$
$$Y_H = Y \qquad \Pi_H = 1 \qquad Z = 1$$
$$\lambda_1^c = Y^{\varphi} \quad \lambda_2^c = \lambda_1^c \quad \lambda_3^c = -\lambda_4^c = \frac{\lambda_5^c}{1-\theta} = 0 \quad \lambda_6^c = \frac{Y^{\varphi}}{1-\theta} \quad \lambda_7^c = 0$$

The fiscal policy problem under no-coordination

The fiscal policy makers maximize the following lagrangian with respect to C_t, G_t, Y_t, $Y_{H,t}$, Z_t, K_t, F_t and $\Pi_{H,t}$:

[38]Namely if even τ is chosen optimally in such a way $\lambda_3 = -\lambda_4 = 0$

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} & \beta^t E_0 \Big\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}Z_t}{A_t}\right)^{\varphi+1} \\
& + \lambda_{1,t}^f \left[Y_t - \left(\frac{P_t}{P_{C,t}}\right)^{-\eta} \left((1-\alpha)C_t + \alpha C_t^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}}{P_{G,t}}\right)^{-\eta} G_t + \nu C_t^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta} \right) \right] \\
& + \lambda_{2,t}^f \left[Y_{H,t} - \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left((1-\psi)Y_t + \psi \left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta} \right) \right] \\
& + \lambda_{3,t}^f \left[K_t - \left(\frac{Y_{H,t}}{A_t}\right)^{\varphi+1} Z_t^{\varphi}(1-\tau)(1+\mu_t)\frac{\varepsilon}{\varepsilon-1} \right] - \lambda_{3,t-1}^f \theta \Pi_{H,t}^{\varepsilon} K_t \\
& + \lambda_{4,t}^f \left[F_t - Y_{H,t}C_t^{-\sigma}\frac{P_t}{P_{C,t}}\frac{P_{H,t}}{P_t} \right] - \lambda_{4,t-1}^f \theta \Pi_{H,t}^{(\varepsilon-1)} F_t \\
& + \lambda_{5,t}^f \left[F_t - K_t \left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{\varepsilon-1}} \right] \\
& + \lambda_{6,t}^f \left[Z_t - \theta Z_{t-1}\Pi_{H,t}^{\varepsilon} - (1-\theta)\left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right] \\
& + \lambda_{7,t}^f \left[\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma} \left(\frac{C_{t-1}^*}{C_t^*}\right)^{-\sigma} \Pi_t^* - \frac{P_{C,t}}{P_t}\frac{P_t}{P_{H,t}}\Pi_{H,t}\frac{P_{H,t-1}}{P_{t-1}}\frac{P_{t-1}}{P_{C,t-1}} \right] \Big\}
\end{aligned}
$$

where $P_t/P_{C,t}$, $P_{G,t}/P_{C,t}$ and $P_{H,t}/P_t$ are determined according (35), (36) and (37) and C_t^*, $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$ are taken as given. According to first order conditions evaluated at the zero inflation symmetric non-stochastic steady state:

$$
\begin{aligned}
& C^{-\sigma} = \lambda_1^f(\delta_1 + \delta_2\frac{\rho}{1-\rho}) + \lambda_2^f(1-\psi)\delta_3\frac{1}{\rho} - \lambda_4^f Y C^{-\sigma}[C^{-1}\sigma + (\omega_4 - 1)] + \lambda_7^f\frac{(1-\beta)}{C}[\sigma - (\omega_4 - 1)] \\
& \chi G^{-\gamma} = \lambda_1^f \\
& \lambda_1^f = \lambda_2^f(1-\psi) \\
& Y^{\varphi} = \lambda_2^f - \lambda_3^f(\varphi+1)Y^{\varphi}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} - \lambda_4^f C^{-\sigma} \\
& Y^{\varphi+1} = -\lambda_3^f\varphi Y^{\varphi+1} + \lambda_6^f(1-\theta) \\
& \lambda_3^f(1-\theta) = \lambda_5^f \\
& \lambda_4^f(1-\theta) = -\lambda_5^f \\
& \lambda_3^f\theta\varepsilon K = -\lambda_4^f\theta(\varepsilon-1)F + \lambda_5^f\frac{\theta}{1-\theta}K - \lambda_7^f
\end{aligned}
$$

If $(1-\tau) = ((1/(1+\mu))(\varepsilon-1)/\varepsilon)(1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]$[39] this system of equations jointly with (38), (44), (45), (52), (53), (55), and (56) can be satisfied by the following

[39]Namely if even τ is chosen to maximize the objective of the fiscal policy maker ensuring $\lambda_3^f = 0$.

solution:

$$
\begin{aligned}
&C^{-\sigma} = (1-\psi)\left[\delta_1 + \delta_2 \frac{G}{C} + \delta_3 \frac{Y}{C}\right] Y^{\varphi} \\
&\chi G^{-\gamma} = (1-\psi)(1-\nu) Y^{\varphi} \\
&Y = C + G \\
&F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}}{1-\theta}(1+\mu)(1-\tau)\frac{\varepsilon}{\varepsilon-1}
\end{aligned}
$$

$$
\begin{aligned}
&Y_H = Y \qquad \Pi_H = 1 \qquad Z = 1 \\
&\lambda_1^f = (1-\psi)Y^{\varphi} \quad \lambda_2^f = Y^{\varphi} \quad \lambda_3^f = -\lambda_4^f = \frac{\lambda_5^f}{1-\theta} = 0 \quad \lambda_6^f = \frac{Y^{\varphi+1}}{1-\theta} \quad \lambda_7^f = 0
\end{aligned}
$$

The monetary policy problem under no-coordination

The monetary policy maker maximizes with respect to C_t^i, Y_t^i, $Y_{H,t}^i$, Z_t^i, K_t^i, F_t^i and $\Pi_{H,t}^i$ for all i and t the following lagragian:

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} \beta^t E_0 \int_0^1 \Bigg\{ & \frac{{C_t^i}^{1-\sigma}}{1-\sigma} + \chi \frac{{G_t^i}^{1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i Z_t^i}{A_t^i}\right)^{\varphi+1} \\
& + \lambda_{1,t}^{i,m}\left[Y_t^i - \left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta}\left((1-\alpha)C_t^i + \alpha {C_t^i}^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}^i}{P_{G,t}^i}\right)^{-\eta} G_t^i + \nu {C_t^i}^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right)\right] \\
& + \lambda_{2,t}^{i,m}\left[Y_{H,t}^i - \left(\frac{P_{H,t}^i}{P_t^i}\right)^{-\eta}\left((1-\psi)Y_t^i + \psi\left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta} {C_t^i}^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right)\right] \\
& + \lambda_{3,t}^{i,m}\left[K_t^i - \left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1} {Z_t^i}^{\varphi}(1-\tau)(1+\mu_t^i)\frac{\varepsilon}{\varepsilon-1}\right] - \lambda_{3,t-1}^{i,m}\theta {\Pi_{H,t}^i}^{\varepsilon} K_t^i \\
& + \lambda_{4,t}^{i,m}\left[F_t^i - Y_{H,t}^i {C_t^i}^{-\sigma}\frac{P_t^i}{P_{C,t}^i}\frac{P_{H,t}^i}{P_t^i}\right] - \lambda_{4,t-1}^{i,m}\theta {\Pi_{H,t}^i}^{(\varepsilon-1)} F_t^i \\
& + \lambda_{5,t}^{i,m}\left[F_t^i - K_t^i\left(\frac{1-\theta {\Pi_{H,t}^i}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right] \\
& + \lambda_{6,t}^{i,m}\left[Z_t^i - \theta Z_{t-1}^i {\Pi^i}^{\varepsilon}_{H,t} - (1-\theta)\left(\frac{1-\theta {\Pi_{H,t}^i}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] \\
& + \lambda_{7,t}^{i,m}\left[\left(\frac{C_t^i}{C_{t-1}^i}\right)^{-\sigma}\left(\frac{C_{t-1}^*}{C_t^*}\right)^{-\sigma}\Pi_t^* - \frac{P_{C,t}^i}{P_t^i}\frac{P_t^i}{P_{H,t}^i}\Pi_{H,t}^i\frac{P_{H,t-1}^i}{P_{t-1}^i}\frac{P_{t-1}^i}{P_{C,t-1}^i}\right]\Bigg\} di
\end{aligned}
$$

where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$, C_t^*, $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$ are determined according (35), (36), (37), (33) and (46) G_t^i is taken as given for all i and t and $Z_{-1} = 1$

. According to the first order conditions evaluated at the zero inflation symmetric non-stochastic steady state:

$$C^{-\sigma} = \lambda_1^m - \lambda_4^m \sigma Y C^{-\sigma-1}$$
$$\lambda_1^m = \lambda_2^m$$
$$Y^{\varphi} = \lambda_2^m - \lambda_3^m(\varphi+1)Y^{\varphi}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} - \lambda_4^m C^{-\sigma}$$
$$Y^{\varphi+1} = -\lambda_3^m \varphi Y^{\varphi+1} + \lambda_6^m(1-\theta)$$
$$\lambda_3^m(1-\theta) = \lambda_5^m$$
$$\lambda_4^m(1-\theta) = -\lambda_5^m$$
$$\lambda_3^m \theta\varepsilon K = -\lambda_4^m \theta(\varepsilon-1)F + \lambda_5^m \frac{\theta}{1-\theta}K$$

It easy to show that if $(1-\tau) = ((1/(1+\mu))(\varepsilon-1)/\varepsilon)(1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]$ this system of equations jointly with (38), (44), (45), (52), (53), (55), and (56) can be satisfied by the following solution:

$$C^{-\sigma} = (1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]Y^{\varphi}$$
$$Y = C + G$$
$$F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}}{1-\theta}(1+\mu)(1-\tau)\frac{\varepsilon}{\varepsilon-1}$$

$$Y_H = Y \qquad \Pi_H = 1 \qquad Z = 1$$
$$\lambda_1^m = Y^{\varphi}\left[\frac{C}{Y}\delta\varphi + \sigma\right] \Big/ \left[\frac{C}{Y}\varphi + \sigma\right] \qquad \lambda_2^m = \lambda_1^m$$
$$\lambda_3^m = -\lambda_4^m = \frac{\lambda_5^m}{1-\theta} = \frac{C}{Y}\frac{(\delta-1)}{\delta} \Big/ \left[\frac{C}{Y}\varphi + \sigma\right] \qquad \lambda_6^m = \frac{Y^{\varphi+1}(1-\varphi\lambda_4^m)}{1-\theta} \qquad \lambda_7^m = 0$$

A purely quadratic approximation of policy makers' objectives

In order to recover the optimal policies we need to approximate up to the second order single country representative agent utility given by (1) in the following way.

First we can approximate the utility derived from private consumption as:

$$\frac{C_t^{1-\sigma}}{1-\sigma} \simeq \frac{C^{1-\sigma}}{1-\sigma} + C^{1-\sigma}(\hat{c}_t + \frac{1}{2}\hat{c}_t^2) - \frac{\sigma}{2}C^{1-\sigma}\hat{c}_t^2 + t.i.p. \tag{95}$$

where $\hat{c}_t$ stands for the log-deviations of private consumption from the steady state[40].

[40]From now this convention will be used: $\hat{x}_t$ represents the log-deviation of X_t from the steady state.

Similarly the utility derived from the consumption of public goods can be approximated:

$$\frac{G_t^{1-\gamma}}{1-\gamma} \simeq \frac{G^{1-\gamma}}{1-\gamma} + G^{1-\gamma}(\hat{g}_t + \frac{1}{2}\hat{g}_t^2) - \frac{\gamma}{2}G^{1-\gamma}\hat{g}_t^2 + t.i.p. \tag{96}$$

The labor disutility can be approximated by taking into account that $N_t = \frac{Y_{H,t}Z_t}{A_t}$ and, as showed by Galí and Monacelli (2008), being $Z_t = \int_0^1 \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} dk$:

$$\hat{z}_t \simeq \frac{\varepsilon}{2} Var_k(p_{H,t}(k)) \tag{97}$$

In words the approximation of Z_t around the symmetric steady state is purely quadratic. Moreover following Woodford (2001, NBER WP8071) it is possible to show that $\sum_{t=0}^{\infty}\beta^t Var_k(p_{H,t}(k)) = \frac{1}{\lambda}\sum_{t=0}^{\infty}\beta^t \pi_{H,t}^2$ with $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Thus:

$$\frac{1}{\varphi+1}\left(\frac{Y_t Z_t}{A_t}\right)^{\varphi+1} \simeq \frac{1}{\varphi+1}Y^{\varphi+1} + Y^{\varphi+1}(\hat{y}_{H,t} + \frac{1}{2}\hat{y}_{H,t}^2) + Y^{\varphi+1}\frac{\varepsilon}{2\lambda}(\pi_{H,t})^2 + \frac{\varphi}{2}Y^{\varphi+1}\hat{y}_{H,t}^2$$
$$-(\varphi+1)Y^{\varphi+1}\hat{y}_{H,t}a_t + t.i.p. \tag{98}$$

The welfare approximation under coordination

Under coordination, at the steady state, the fiscal authority chooses to produce the efficient level of public goods. Therefore $C^{-\sigma} = \chi G^{-\gamma} = Y^{\varphi}$ which implies that the second order approximation of the average union welfare can be rewritten as:

$$\sum_{t=0}^{\infty}\beta^t Y^{\varphi+1} E_0 \int_0^1 \left[\hat{s}_t^{i'} z_s - \frac{1}{2}\hat{s}_t^{i'} Z_{s,s}\hat{s}_t^i + \hat{s}_t^{i'} Z_{s,a}\hat{a}_t^i\right] + t.i.p. \tag{99}$$

where

$$\hat{s}_t' \equiv [\hat{y}_{H,t}^i,\ \hat{g}_t^i,\ \hat{c}_t^i,\ \pi_{H,t}^i] \qquad z_s^{'} \equiv [-1,\ \rho,\ (1-\rho),\ 0]$$

$$Z_{s,s} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 & 0 \\ 0 & (\gamma-1)(1-\rho) & 0 & 0 \\ 0 & 0 & (\sigma-1)\rho & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix}$$

$$Z_{s,a} \equiv \begin{bmatrix} (1+\varphi) \\ 0 \\ 0 \end{bmatrix}$$

Again it is possible to substitute the linear quadratic terms of (95) by using the second order approximation of the resource constraints namely:

$$0 \simeq -\int_0^1 \hat{y}_t^i di - \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \int_0^1 \hat{s}_t^i di'\ h_s + \frac{1}{2}\int_0^1 \hat{s}_t^{i'} H_{s,s}\hat{s}_t^i di + \frac{1}{2}\int_0^1 \hat{s}_t^i di' H_{S,S}\int_0^1 \hat{s}_t^i di + t.i.p. \quad (100)$$

$$0 \simeq \int_0^1 \hat{s}_t^i di'\ p_s + \int_0^1 \hat{y}_t^i di + \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \frac{1}{2}\int_0^1 \hat{s}_t^{i'} P_{s,s}\hat{s}_t^i di + \frac{1}{2}\int_0^1 \hat{s}_t^i di P_{S,S}\int_0^1 \hat{s}_t^i di + \int_0^1 \hat{y}_t^i P_{y,s}\hat{s}_t^i di$$

$$+\int_0^1 \hat{y}_t^i di P_{Y,S}\int_0^1 \hat{s}_t^i di + t.i.p. \quad (101)$$

where

$$h_s' \equiv [0,\ (1-\rho),\ \rho,\ 0]$$

$$H_{s,s} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1-\rho) & \xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & \xi\nu(1-\nu)(1-\rho) & \rho+\omega_1\rho+\omega_2(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_{S,S} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & -\xi\nu(1-\nu)(1-\rho) & -\omega_1\rho-\omega_2(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$p_s' \equiv [-1,\ 0,\ 0,\ 0] \qquad P_{y,s} \equiv \begin{bmatrix} 0 & 0 & \xi\psi & 0 \end{bmatrix} \qquad P_{Y,S} \equiv \begin{bmatrix} 0 & 0 & -\xi\psi & 0 \end{bmatrix}$$

$$P_{s,s} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad P_{S,S} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$\xi \equiv \frac{\eta\sigma}{1-\alpha}$$

$$\omega_1 \equiv \frac{\alpha\,\eta\,\sigma\ (\sigma-(1-\alpha)\ \alpha\ (1-\eta\,\sigma))}{(1-\alpha)^2}$$

$$\omega_2 \equiv \frac{\eta\,\nu\,\sigma\ ((\nu-1)+(\sigma-1)-(1-2\,\eta)\ (1-\nu)\ \nu\,\sigma-\alpha\ (\nu-2)\ (1+(1-\eta)\ \sigma)-(1-\eta)\ \sigma\nu)}{(1-\alpha)^2}$$

$$\omega_3 \equiv -\left(\frac{\eta\,\sigma\,\psi\ ((1-\alpha-\alpha\ (1-\eta)\ \sigma)\ (1-\psi)\ (2-\psi)-\sigma\ (1+\eta\ (1-\psi)\ \psi))}{(1-\alpha)^2\,(1-\psi)^2}\right)$$

Given (116) and (117) it is easy to show that:

$$0 \simeq \int_0^1 \hat{s}_t^i di'\ r_s + \frac{1}{2}\int_0^1 \hat{s}_t^{i'} R_{s,s}\hat{s}_t^i di + \frac{1}{2}\int_0^1 \hat{s}_t^i di' R_{S,S}\int_0^1 \hat{s}_t^i di + t.i.p. \quad (102)$$

where

$$r_s \equiv p_s + h_s \qquad R_{s,s} \equiv P_{s,s} + H_{s,s} + h_y P_{y,s} + P'_{y,s} h'_y$$

$$R_{S,S} \equiv \ P_{S,S} + H_{S,S} + h_Y P_{y,s} + P'_{y,s} h'_Y + h_s P_{Y,S} + P'_{Y,S} h'_s \tag{103}$$

and

$h_y^{'} \equiv [0,\ (1-\nu)(1-\rho),\ \delta_1\rho + \delta_2(1-\rho),\ 0]$ $\qquad$ $h_Y^{'} \equiv [0,\ \nu(1-\rho), \rho - \delta_1\rho - \delta_2(1-\rho),\ 0]$

where $\varsigma_1 \equiv \xi(\psi+\nu)$ $\varsigma_3 \equiv \rho\omega_1 + (1-\rho)\omega_2 + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$

Given that

$$z_s = r_s \tag{104}$$

under coordination, the second order approximation to the average union welfare can be rewritten as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[-\frac{1}{2}\int_0^1 \hat{s}_t^{i'}\Omega_{s,s}\hat{s}_t^i di - \frac{1}{2}\int_0^1 \hat{s}_t^i di'\Omega_{S,S}\int_0^1 \hat{s}_t^i di + \int_0^1 \hat{s}_t^{i'}\Omega_{s,a}\hat{a}_t^i di + \int_0^1 \hat{s}_t^i di'\Omega_{S,A}\int_0^1 \hat{a}_t^i di\Big] + t.i.p. \tag{105}$$

where

$$\Omega_{s,s} \equiv Z_{s,s} + R_{s,s} \qquad \Omega_{S,S} \equiv R_{S,S}$$

$$\Omega_{s,a} \equiv Z_{s,a}$$

are equal to:

$$\Omega_{s,s} = \begin{bmatrix} \varphi & 0 & 0 & 0 \\ 0 & \gamma(1-\rho) & (1-\rho)(1-\nu)\varsigma_1 & 0 \\ 0 & (1-\rho)(1-\nu)\varsigma_1 & \sigma\rho + \varsigma_3 & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix}$$

$$\Omega_{S,S} = \begin{bmatrix} 0 & -(1-\rho)(1-\nu)\varsigma_1 & 0 \\ -(1-\rho)(1-\nu)\varsigma_1 & -\varsigma_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The welfare for the fiscal authority under no-coordination

By combining (95),(96) and (98) and considering that at the steady state $C^{-\sigma} = (1-\psi)\delta Y^{\varphi}$and $\chi G^{-\gamma} = (1-\psi)(1-\nu)Y^{\varphi}$ the second order approximation of single country representative agent welfare can be written in matrix notation as:

$$\sum_{t=0}^{\infty}\beta^t Y^{\varphi+1} E_0\left[\hat{s}_t' w_s - \frac{1}{2}\hat{s}_t' W_{s,s}\hat{s}_t + \hat{s}_t' W_{s,e}\hat{e}_t\right] + t.i.f.p. \tag{106}$$

where

$\hat{s}_t' \equiv [\hat{y}_{H,t},\ \hat{g}_t,\ \hat{c}_t,\ \pi_{H,t}] \qquad w_s^{'} \equiv [-1,\ (1-\psi)(1-\nu)(1-\rho),\ (1-\psi)\delta\rho,\ 0] \qquad \hat{e}_t^{'} \equiv [\hat{y}_t^*,\ \hat{g}_t^*,\ \hat{c}_t^*,\ a_t]$

$$W_{s,s} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 & 0 \\ 0 & (\gamma-1)(1-\psi)(1-\nu)(1-\rho) & 0 & 0 \\ 0 & 0 & (\sigma-1)(1-\psi)\delta\rho & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix}$$

$$W_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & (1+\varphi) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and with $\hat{y}_t^* \equiv \int_0^1 \hat{y}_t^j dj$, $\hat{g}_t^* \equiv \int_0^1 \hat{g}_t^j dj$ and $\hat{c}_t^* \equiv \int_0^1 \hat{c}_t^j dj$. This approximation can be written in purely quadratic way by using the second order approximation of the single country market clearing conditions (44) and (45). In particular notice that the second order approximation of these constraints can be read as:

$$0 \simeq \left[-\hat{y}_t - \tfrac{1}{2}\hat{y}_t^2 + \hat{s}_t' f_s - \hat{e}_t' f_e + \tfrac{1}{2}\hat{s}_t' F_{s,s}\hat{s}_t - \hat{s}_t' F_{s,e} e_t\right] + s.o.t.i.f.p. \tag{107}$$

$$0 \simeq \left[\hat{s}_t'\iota_s - \hat{e}_t'\iota_e + \hat{y}_t\iota_y + \tfrac{1}{2}\hat{y}_t^2\iota_y + \tfrac{1}{2}\hat{s}_t^{'} I_{s,s}\hat{s}_t - \hat{s}_t' I_{s,e}\hat{e}_t + \hat{y}_t I_{y,s}\hat{s}_t - \hat{y}_t I_{y,e}\hat{e}_t\right] + s.o.t.i.f.p. \tag{108}$$

where

$f_s^{'} \equiv [0,\ (1-\nu)(1-\rho),\ \delta_1\rho + \delta_2(1-\rho),\ 0] \qquad f_e^{'} \equiv [0,\ -\nu(1-\rho),\ -\rho + (\delta_1\rho + \delta_2(1-\rho)),\ 0]$

$$F_{s,s} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1-\nu)(1-\rho) & \xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & \xi\nu(1-\nu)(1-\rho) & \delta_1\rho + \delta_2(1-\rho) + \omega_1\rho + \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & -\xi\nu(1-\rho) & \omega_1\rho + \omega_2(1-\rho) + \xi\nu(2-\nu)(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\iota_s^{'} \equiv [-1,\ 0,\ (1-\psi)\delta_3,\ 0] \qquad \iota_e^{'} \equiv [-\psi,\ 0,\ (1-\psi)\delta_3,\ 0] \qquad \iota_y \equiv [(1-\psi)]$

$$I_{s,s} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\psi)\delta_3 + \omega_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad I_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\xi\psi}{(1-\psi)} & 0 & \omega_3 + \frac{\xi\psi(2-\psi)}{(1-\psi)} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I_{y,s} \equiv \begin{bmatrix} 0 & 0 & \xi\psi & 0 \end{bmatrix} \qquad I_{y,e} \equiv \begin{bmatrix} 0 & 0 & \xi\psi & 0 \end{bmatrix}$$

Given (107), (108) an be rewritten as:

$$0 \simeq \hat{s}_t^{'}\left(\iota_s + (1-\psi)f_s\right) - \hat{e}_t^{'}(\iota_e + (1-\psi)f_e) + \frac{1}{2}\hat{s}_t^{'}\left(I_{s,s} + (1-\psi)F_{s,s} + f_s I_{y,s} + I_{y,s}' f_s'\right)\hat{s}_t$$
$$-\hat{s}_t^{'}\left[I_{s,e} + (1-\psi)F_{s,e} + f_s I_{y,e} + f_e I_{y,s}\right]\hat{e}_t + s.o.t.i.f.p. \tag{109}$$

Again thanks to conditions (74), (75) and (76) it follows that:

$$w_s = \iota_s + (1-\psi) f_s \tag{110}$$

Therefore by using (109), (106) can be approximated as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[-\frac{1}{2}s_t^{'}\Omega_{s,s}s_t + s_t^{'}\Omega_{s,e}e_t\Big] + t.i.f.p. \tag{111}$$

which is purely quadratic and where $\Omega_{s,s} \equiv W_{s,s} + I_{s,s} + (1-\psi)F_{s,s} + f_s I_{y,s} + I_{y,s}^{\prime} f_s^{\prime}$ and $\Omega_{s,e} \equiv W_{s,e} + I_{s,e} + (1-\psi)F_{s,e} + f_s I_{y,e} + f_e I_{y,s}$ are respectively equal to:

$$\begin{bmatrix} \varphi & 0 & 0 & 0 \\ 0 & \gamma(1-\rho)(1-\nu)(1-\psi) & (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi) & 0 \\ 0 & (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi) & (1-\psi)\rho\sigma\delta + \varsigma_2 & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix} \tag{112}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1+\varphi \\ 0 & 0 & (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi) & 0 \\ -(1-\psi)\delta_3 & -(1-\rho)(1-\psi)\delta_2 + (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi) & (1-\psi)((1-\rho)\delta_2 + \delta_3) + \varsigma_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{113}$$

with $\delta \equiv \delta_1 + \frac{(1-\rho)}{\rho}\delta_2 + \frac{1}{\rho}\delta_3$, $\varsigma_1 \equiv \xi(\nu+\psi)$ $\varsigma_2 \equiv (1-\psi)(\omega_1\rho + \omega_2(1-\rho)) + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$.

The welfare approximation for the monetary authority

The central bank of the monetary union maximizes:

$$\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} + \chi\frac{G_t^{i\,1-\gamma}}{1-\gamma} - \frac{N_t^{i\,\varphi+1}}{\varphi+1}\right] di \quad 0<\beta<1 \tag{114}$$

By combining (95) and (98) and given that $C^{-\sigma} = (1-\psi)\delta Y^{\varphi}$, the second order approximation of (114) can be written as:

$$\sum_{t=0}^{\infty}\beta^t Y^{\varphi+1} E_0 \int_0^1 \left[\hat{l}_t^{i\prime} w_l - \frac{1}{2}\hat{l}_t^{i\prime} W_{l,l}\hat{l}_t^i + \hat{l}_t^{i\prime} W_{l,u}\hat{u}_t^i\right] di + t.i.m.p. \tag{115}$$

where

$$\hat{l}_t^{i\prime} \equiv \left[\hat{y}_{H,t}^i,\ \hat{c}_t^i,\ \hat{\pi}_{H,t}^i\right] \qquad \hat{u}_t^{i\prime} \equiv \left[\hat{g}_t^i,\ a_t^i, \mu_t^i\right] \qquad w_l^{\prime} \equiv [-1,\ (1-\psi)\delta\rho,\ 0]$$

$$W_{l,l} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 \\ 0 & (\sigma-1)(1-\psi)\delta\rho & 0 \\ 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix} \qquad W_{l,u} \equiv \begin{bmatrix} 0 & (\varphi+1) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $t.i.m.p.$ stands for terms independent of monetary policy inclusive of the government expenditure. In order to express that approximation in a purely quadratic way, it is necessary to recover the second order approximations of (44), (45), (55), (50) and (51). By integrating the first two approximation we obtain:

$$0 \simeq -\int_0^1 \hat{y}_t^i di - \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \int_0^1 \hat{l}_t^i di' \ f_l - \int_0^1 \hat{u}_t^i di' \ f_u + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} F_{l,l} \hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i dj' F_{L,L} \int_0^1 \hat{l}_t^i di$$
$$-\int_0^1 \hat{l}_t^{i'} F_{l,u} \hat{u}_t^i di - \int_0^1 \hat{l}_t^{i'} di F_{L,U} \int_0^1 \hat{u}_t^i di + s.o.t.i.m.p. \tag{116}$$

$$0 \simeq \int_0^1 \hat{l}_t^i di' \ \iota_l + \int_0^1 \hat{y}_t^i di + \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} I_{l,l} \hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di' I_{L,L} \int_0^1 \hat{l}_t^i di + \int_0^1 \hat{y}_t^i I_{y,l} \hat{l}_t^i di$$
$$+\int_0^1 \hat{y}_t^i di I_{Y,L} \int_0^1 \hat{l}_t^i di + s.o.t.i.m.p. \tag{117}$$

where

$$f_l' \equiv [0,\ \rho,\ 0] \qquad f_u' \equiv [-(1-\rho),\ 0, 0]$$

$$F_{l,l} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho + \omega_1\rho + \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad F_{l,u} \equiv \begin{bmatrix} 0 & 0 & 0 \\ -\xi\nu(1-\nu)(1-\rho) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$F_{L,L} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\omega_1\rho - \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad F_{L,U} \equiv \begin{bmatrix} 0 & 0 & 0 \\ \xi\nu(1-\nu)(1-\rho) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\iota_l' \equiv [-1,\ 0,\ 0] \qquad I_{y,l} \equiv \begin{bmatrix} 0 & \xi\psi & 0 \end{bmatrix} \qquad I_{Y,L} \equiv \begin{bmatrix} 0 & -\xi\psi & 0 \end{bmatrix}$$

$$I_{l,l} \equiv \begin{bmatrix} -1 & 0 & 0 \\ 0 & \omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad I_{L,L} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Given (116) and (117) it is easy to show that:

$$0 \simeq \int_0^1 \hat{l}_t^i di' \ r_l - \int_0^1 \hat{u}_t^i di' \ r_u + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} R_{l,l} \hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di' R_{L,L} \int_0^1 \hat{l}_t^i di$$
$$-\int_0^1 \hat{l}_t^{i'} R_{l,u} u_t^i di - \int_0^1 \hat{l}_t^{i'} di R_{L,U} \int_0^1 u_t^i di + s.o.t.i.m.p. \tag{118}$$

where

$$r_l \equiv \iota_l + f_l \qquad r_u \equiv f_u$$

$$R_{l,l} \equiv I_{l,l} + F_{l,l} + f_y I_{y,l} + I_{y,l}' f_y' \qquad R_{L,L} \equiv I_{L,L} + F_{L,L} + f_Y I_{y,l} + I_{y,l}' f_Y + f_l I_{Y,L} + I_{Y,L}' f_l'$$
$$R_{l,u} \equiv F_{l,u} + I_{y,l}' f_g' \qquad R_{L,U} \equiv F_{L,U} + f_G I_{y,l} + I_{Y,L}' f_u' \tag{119}$$

and

$$f_y' \equiv [0,\ \delta_1\rho + \delta_2(1-\rho),\ 0] \qquad f_Y' \equiv [0,\ \rho - \delta_1\rho - \delta_2(1-\rho),\ 0]$$

$$f_g' \equiv [-(1-\nu)(1-\rho),\ 0] \qquad f_G' \equiv [-\nu(1-\rho),\ 0]$$

By combining the second order approximation of the (55), (52) and (53) as in Benigno and Woodford (2005), we obtain the following condition:

$$V_0 = \frac{1-\theta}{\theta}(1-\beta\theta)\sum_{t=0}^{\infty}\beta^t E_0\Big[\int_0^1 \hat{l}_t^i di' v_l - \int_0^1 \hat{u}_t^{i\prime} v_u + \frac{1}{2}\int_0^1 \hat{l}_t^{i\prime} V_{l,l}\hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di' V_{L,L}\int_0^1 \hat{l}_t^i di - \int_0^1 \hat{l}_t^{i\prime} V_{l,u}\hat{u}_t^i di\Big] + s.o.t.i.m.p. \quad (120)$$

where

$$v_l' \equiv [\varphi,\ \sigma,\ 0] \qquad v_u' \equiv [0,\ (\varphi+1),\ -1]$$

$$V_{l,l} \equiv \begin{bmatrix} \varphi(\varphi+2) & \omega_4 & 0 \\ \omega_4 & -\omega_4^2+\omega_5 & 0 \\ 0 & 0 & \frac{\varepsilon(\varphi+1)}{\lambda} \end{bmatrix} \qquad V_{l,u} \equiv \begin{bmatrix} 0 & (\varphi+1)^2 & -(\varphi+1) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_{L,L} \equiv \begin{bmatrix} 0 & \sigma-\omega_4 & 0 \\ \sigma-\omega_4 & -\sigma^2+\omega_4^2-\omega_5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with

$$\omega_5 \equiv -\frac{\sigma\,\psi\,(-1+(1-\eta)\,\sigma\,(1+\alpha\,(1-\psi))+\alpha\,(1-\psi)+(1-\sigma)\,\psi)}{(1-\alpha)^2\,(1-\psi)^2} + \frac{\alpha\,\sigma\,(1-\alpha\,(1-\sigma)-(1-\eta)\,\sigma)}{(1-\alpha)^2}$$

$$\omega_4 \equiv \frac{\sigma}{(1-\alpha)\,(1-\psi)} \qquad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

Conditions (118) and (121) allow to substitute the linear term of the union welfare approximation with purely quadratic terms. In fact given these conditions:

$$0 \simeq Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\int_0^1 \hat{l}_t^i di'(\zeta_1 r_l+\zeta_2 v_l) + \frac{1}{2}\int_0^1 \hat{l}_t^{i\prime}(\zeta_1 R_{l,l}+\zeta_2 V_{l,l})\hat{l}_t^i di - \int_0^1 \hat{l}_t^{i\prime}(\zeta_1 R_{l,u}+\zeta_2 V_{l,u})\hat{u}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di'(\zeta_1 R_{L,L}+\zeta_2 V_{L,L})\int_0^1 \hat{l}_t^i di - \int_0^1 \hat{l}_t^i di'(\zeta_1 R_{L,U})\int_0^1 \hat{u}_t^i di\Big] + t.i.m.p. \quad (121)$$

where $\zeta_1 \equiv \frac{(1-\psi)\delta\varphi\rho+\sigma}{\varphi\rho+\sigma}$ and $\zeta_2 \equiv \frac{((1-\psi)\delta-1)\rho}{\varphi\rho+\sigma}$. It is easy to show that:

$$w_l = \zeta_1 r_l + \zeta_2 v_l \quad (122)$$

Hence we can write the second order approximation of union welfare as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[-\frac{1}{2}\int_0^1 \hat{l}_t^{i\prime}\Omega_{l,l}\hat{l}_t^i di - \frac{1}{2}\int_0^1 \hat{l}_t^i di'\Omega_{L,L}\int_0^1 \hat{l}_t^i di + \int_0^1 \hat{l}_t^{i\prime}\Omega_{l,u}\hat{u}_t^i di + \int_0^1 \hat{l}_t^i di'\Omega_{L,U}\int_0^1 \hat{u}_t^i di\Big] + t.i.m.p. \quad (123)$$

where

$$
\Omega_{l,l} \equiv W_{l,l} + \zeta_1 R_{l,l} + \zeta_2 V_{l,l} \qquad \Omega_{L,L} \equiv \zeta_1 R_{L,L} + \zeta_2 V_{L,L}
$$

$$
\Omega_{l,u} \equiv W_{l,u} + \zeta_1 R_{l,u} + \zeta_2 V_{l,u} \qquad \Omega_{L,U} \equiv \zeta_1 R_{L,U} \tag{124}
$$

are equal to:

$$
\Omega_{l,l} = \begin{bmatrix} \varphi\zeta_3 & \zeta_2\omega_4 & 0 \\ \zeta_2\omega_4 & \delta(\sigma-1)(1-\psi)\rho + \zeta_1(\rho+\varsigma_3) + \zeta_2(\omega_5-\omega_4^2) & 0 \\ 0 & 0 & \frac{\varepsilon\zeta_3}{\lambda} \end{bmatrix}
$$

$$
\Omega_{L,L} = \begin{bmatrix} 0 & \zeta_2(\sigma-\omega_4) & 0 \\ \zeta_2(\sigma-\omega_4) & -\zeta_1\varsigma_3 - \zeta_2(\sigma^2+\omega_5-\omega_4^2) & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\Omega_{l,u} = \begin{bmatrix} 0 & (\varphi+1)\zeta_3 & -(\varphi+1)\zeta_2 \\ -\zeta_1(1-\nu)(1-\rho)\varsigma_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \Omega_{L,U} = \begin{bmatrix} 0 & 0 & 0 \\ \zeta_1(1-\nu)(1-\rho)\varsigma_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

with $\zeta_3 \equiv 1 + (\varphi+1)\zeta_2$ and $\varsigma_3 \equiv \rho\omega_1 + (1-\rho)\omega_2 + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$.